Knowledge Organiser: Year 9 Maths; Simultaneous Equations (Part 1)

Algebra — Simplifying

Algebra really terrifies so many people. But honestly, it's not that bad.
Make sure you understand and learn these basics for dealing with algebraic expressions.

Terms (Grade 2)

Before you can do anything else with algebra, you must understand what a term is:

A TERM IS A COLLECTION OF NUMBERS, LETTERS AND BRACKETS, ALL MULTIPLIED/DIVIDED TOGETHER

Terms are separated by + and – signs. Every term has a + or – attached to the front of it.

If there's no sign in front of the first term, it means there's an invisible + sign.

$4x^2$ + 5x – 2y + 6y² + 4

'x^2' term, 'x' term, 'y' term, 'y^2' term, 'number' term

Simplifying or 'Collecting Like Terms' (Grade 2)

To simplify an algebraic expression made up of all the same terms, just add or subtract them.

EXAMPLES:

1. Simplify q + q + q + q + q
   Just add up all the q's:
   q + q + q + q + q = 5q
   'q' just means '1q'.

2. Simplify 4t + 5t – 2t
   Again, just combine the terms — don't forget there's a '–' before the 2t:
   4t + 5t – 2t = 7t

If you have a mixture of different terms, it's a bit more tricky. To simplify an algebraic expression like this, you combine 'like terms' (e.g. all the x terms, all the y terms, all the number terms etc.).

EXAMPLE: Simplify $2x - 4 + 5x + 6$

Invisible + sign — number terms — x-terms

$2x \; -4 \; +5x \; +6 \; = \; \underbrace{2x \; +5x}_{7x} \; \underbrace{-4 \; +6}_{+2} \; = 7x + 2$

1) Put bubbles round each term — be sure you capture the +/– sign in front of each.
2) Then you can move the bubbles into the best order so that like terms are together.
3) Combine like terms.

EXAMPLE: Simplify $4 + \sqrt{3} - 1 + 3\sqrt{3}$ (Grade 5)

Don't be put off by the $\sqrt{3}$-terms. Just treat them like x-terms — don't combine them with the number terms.

Invisible + sign — $\sqrt{3}$-terms — number-terms

$4 \; +\sqrt{3} \; -1 \; +3\sqrt{3} = \underbrace{4 \; -1}_{3} \; \underbrace{+\sqrt{3} \; +3\sqrt{3}}_{+4\sqrt{3}} = 3 + 4\sqrt{3}$

Multiplying Brackets (Grade 3)

The key thing to remember about multiplying brackets is that the thing outside the brackets multiplies each separate term inside the brackets.

EXAMPLE: Expand the following:

a) $3(2x + 5)$
$= (3 \times 2x) + (3 \times 5)$
$= 6x + 15$

b) $-4(3y - 2)$
$= (-4 \times 3y) + (-4 \times -2)$
$= -12y + 8$

c) $2e(e - 4)$
$= (2e \times e) + (2e \times -4)$
$= 2e^2 - 8e$

EXAMPLE: Expand $x(2x + 1) + y(y - 4) + 3x(y + 2)$

1) Expand each bracket separately.
$x(2x + 1) \; + \; y(y - 4) \; + \; 3x(y + 2)$
$= 2x^2 + x \; + \; y^2 - 4y \; + \; 3xy + 6x$

2) Group together like terms.
$= 2x^2 \; + x + 6x \; + 3xy \; + y^2 \; - 4y$

3) Simplify the expression.
$= 2x^2 + 7x + 3xy + y^2 - 4y$

Solving Equations

'Solving equations' basically means 'find the value of x (or whatever letter is used) that makes the equation true'. To do this, you usually have to rearrange the equation to get x on its own.

The 'proper' way to solve equations is to keep rearranging them until you end up with 'x = ' on one side. There are a few important points to remember when rearranging.

Golden Rules

1) Always do the SAME thing to both sides of the equation.
2) To get rid of something, do the opposite.
   The opposite of + is – and the opposite of – is +.
   The opposite of × is ÷ and the opposite of ÷ is ×.
3) Keep going until you have a letter on its own.

EXAMPLES:

1. Solve $x + 7 = 11$.
   The opposite of +7 is –7
   $x + 7 = 11$
   (–7) $x + 7 - 7 = 11 - 7$
   $x = 4$
   This means 'take away 7 from both sides'.

2. Solve $x - 3 = 7$.
   The opposite of –3 is +3
   $x - 3 = 7$
   (+3) $x - 3 + 3 = 7 + 3$
   $x = 10$

3. Solve $5x = 15$.
   5x means 5 × x, so do the opposite — divide both sides by 5
   $5x = 15$
   (÷5) $5x \div 5 = 15 \div 5$
   $x = 3$

4. Solve $\frac{x}{3} = 2$.
   $\frac{x}{3}$ means x ÷ 3, so do the opposite — multiply both sides by 3
   $\frac{x}{3} = 2$
   (×3) $\frac{x}{3} \times 3 = 2 \times 3$
   $x = 6$

Knowledge Organiser: Year 9 Maths; Simultaneous Equations (Part 2)

Solving Equations

You're not done with solving equations yet — not by a long shot. This is where it gets really fun*.

Two-Step Equations (Grade 3)

If you come across an equation like $4x + 3 = 19$ (where there's an x-term and a number on the same side), use the methods from the previous page to solve it — just do it in two steps:

1) Add or subtract the number first. 2) Multiply or divide to get 'x = '.

EXAMPLE: Solve the equation $4x - 3 = 17$.

$4x - 3 = 17$ — The opposite of -3 is $+3$, so add 3 to both sides.

$(+3)$ $4x - 3 + 3 = 17 + 3$

$4x = 20$ — The opposite of $\times 4$ is $\div 4$, so divide both sides by 4.

$(\div 4)$ $4x \div 4 = 20 \div 4$

$x = 5$

Equations with an 'x' on Both Sides (Grade 3)

For equations like $2x + 3 = x + 7$ (where there's an x-term on each side), you have to:

1) Get all the x's on one side and all the numbers on the other.
2) Multiply or divide to get 'x = '.

EXAMPLE: Solve the equation $3x + 5 = 5x + 7$.

$3x + 5 = 5x + 7$ — To get the x's on only one side, subtract $3x$ from each side.

$(-3x)$ $3x + 5 - 3x = 5x + 7 - 3x$

$5 = 2x + 7$ — Now subtract 7 to get the numbers on the other side.

(-7) $5 - 7 = 2x + 7 - 7$

$-2 = 2x$ — The opposite of $\times 2$ is $\div 2$, so divide both sides by 2.

$(\div 2)$ $-2 \div 2 = 2x \div 2$

$-1 = x$

Don't be put off by the fact that the x ends up on the right, not the left — $-1 = x$ is exactly the same as $x = -1$.

Equations with Brackets (Grade 4)

If the equation has brackets in, you have to multiply out the brackets (see p.26) before solving it as above.

EXAMPLE: Solve the equation $5x + 3 = 4(x + 2)$.

$5x + 3 = 4(x + 2)$ — Multiply out the brackets.

$5x + 3 = 4x + 8$ — To get the x's on only one side, subtract $4x$ from each side.

$(-4x)$ $5x + 3 - 4x = 4x + 8 - 4x$

$x + 3 = 8$ — The opposite of $+3$ is -3, so subtract 3 from each side.

(-3) $x + 3 - 3 = 8 - 3$

$x = 5$

Simultaneous Equations

Simultaneous equations might sound a bit scary, but they're just a pair of equations that you have to solve at the same time. You have to find values of x and y that work in both equations.

Six Steps for Simultaneous Equations (Grade 5)

EXAMPLE: Solve the simultaneous equations $2x + 4y = 6$
$4x + 3y = -3$

Both your equations should be in the form $ax + by = c$, where a, b and c are numbers.

1. Label your equations ① and ②.

$2x + 4y = 6$ — ①

$4x + 3y = -3$ — ②

2. Match up the numbers in front of either the x's or y's in both equations. You may need to multiply one or both equations by a suitable number. Relabel the equations ③ and ④ if you need to change them.

① × 2: $4x + 8y = 12$ — ③

$4x + 3y = -3$ — ②

You don't need to change equation 2 for this example.

3. Add or subtract the two equations to eliminate the terms with the same number in front.

③ − ②: $4x + 8y = 12$

$-\ 4x + 3y = -3$

$0x + 5y = 15$

If the numbers have the same sign (both +ve or both −ve) then subtract. If the numbers have opposite signs (one +ve and one −ve) then add.

4. Solve the resulting equation.

$5y = 15 \Rightarrow y = 3$

5. Substitute the value you've found back into equation ① and solve it.

Sub $y = 3$ into ①: $2x + (4 \times 3) = 6 \Rightarrow 2x + 12 = 6 \Rightarrow 2x = -6 \Rightarrow x = -3$

6. Substitute both these values into equation ② to make sure it works. If it doesn't then you've done something wrong and you'll have to do it all again.

Sub x and y into ②: $(4 \times -3) + (3 \times 3) = -12 + 9 = -3$, which is right, so it's worked.

So the solutions are: $x = -3$, $y = 3$

How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.
I will also use these at home to complete my own independent learning and revision of these key topics.

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