Knowledge Organiser: Year 9 Maths; Pythagoras and Trigonometry (Part 1)

Pythagoras' Theorem

Once upon a time there lived a clever chap called Pythagoras. He came up with a clever theorem...

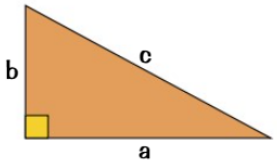
Pythagoras' Theorem is Used on Right-Angled Triangles

(GRADE 4)

Pythagoras' theorem only works for RIGHT-ANGLED TRIANGLES.
It uses two sides to find the third side.

The formula for Pythagoras' theorem is: $a^2 + b^2 = c^2$

short sides, long side



The trouble is, the formula can be quite difficult to use. Instead, it's a lot better to just remember these three simple steps, which work every time:

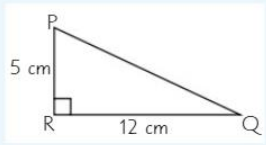
1) SQUARE THEM — SQUARE THE TWO NUMBERS that you are given, (use the x^2 button if you've got your calculator.)

2) ADD or SUBTRACT — To find the longest side, ADD the two squared numbers. $a^2 + b^2 = c^2$
To find a shorter side, SUBTRACT the smaller from the larger. $c^2 - b^2 = a^2$

3) SQUARE ROOT — Once you've got your answer, take the SQUARE ROOT (use the $\sqrt{}$ button on your calculator). $c = \sqrt{a^2+b^2}$, $a = \sqrt{c^2-b^2}$

EXAMPLES:

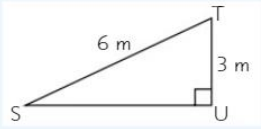
1. Find the length of side PQ in this triangle.



1) Square them: $a^2 = 5^2 = 25$, $b^2 = 12^2 = 144$
2) You want to find the longest side, so ADD: $a^2 + b^2 = c^2$, $25 + 144 = 169$
3) Square root: $c = \sqrt{169} = 13$ cm

Always check the answer's sensible — 13 cm is longer than the other two sides, but not too much longer, so it seems OK.

2. Find the length of SU to 1 decimal place.



1) Square them: $b^2 = 3^2 = 9$, $c^2 = 6^2 = 36$
2) You want to find a shorter side, so SUBTRACT: $c^2 - b^2 = a^2$, $36 - 9 = 27$
3) Square root: $a = \sqrt{27} = 5.196...$
$= 5.2$ m (to 1 d.p.)

Check the answer is sensible — yes, it's a bit shorter than the longest side.

Trigonometry — Sin, Cos, Tan

Trigonometry — it's clever stuff. The three trig formulas are used on right-angled triangles to:
a) find an unknown side if you know a side and an angle, or b) find an angle if you know two lengths.

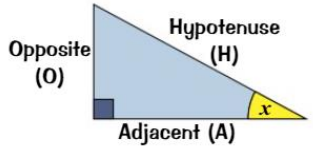
The 3 Trigonometry Formulas

(GRADE 5)

There are three basic trig formulas — each one links two sides and an angle of a right-angled triangle.

$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan x = \frac{\text{Opposite}}{\text{Adjacent}}$

- The Hypotenuse is the LONGEST SIDE.
- The Opposite is the side OPPOSITE the angle being used (x).
- The Adjacent is the (other) side NEXT TO the angle being used.



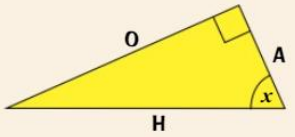
Formula Triangles Make Things Easier

(GRADE 5)

There's more about formula triangles on p.69 if you need to jog your memory.

A great way to tackle trig questions is to convert the formulas into formula triangles.
Then you can use the same method every time, no matter which side or angle is being asked for.

1) Label the three sides O, A and H (Opposite, Adjacent and Hypotenuse).
2) Write down 'SOH CAH TOA'.
3) Decide which two sides are involved: O,H A,H or O,A and choose SOH, CAH or TOA accordingly.



4) Turn the one you choose into a FORMULA TRIANGLE:

In the formula triangles, S represents sin x, C is cos x, and T is tan x.

5) Cover up the thing you want to find with your finger, and write down whatever is left showing.
6) Stick in the numbers and work it out using the sin, cos and tan buttons on your calculator.

If you're finding an angle, you'll need to add an extra step to find the inverse. See next page.

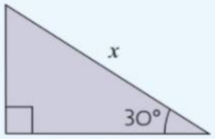
Knowledge Organiser: Year 9 Maths; Pythagoras and Trigonometry (Part 2)

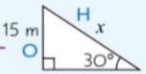
Trigonometry — Examples

Here are some lovely examples using the method from the previous page to help you through the trials of trig.

Examples:

1 Find the length of x in the triangle to the right.



1) Label the sides — 

2) Write down — SOH CAH TOA

3) O and H involved

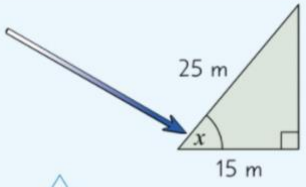
4) Write down the formula triangle — 

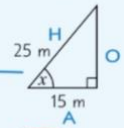
5) You want H so cover it up to give — $H = O \div S$

6) Put in the numbers — $x = \frac{15}{\sin 30°} = \frac{15}{0.5} = 30\text{ m}$

15 ÷ sin 30 =

2 Find the angle x in the triangle to the right.



1) Label the sides — 

2) Write down — SOH CAH TOA

3) A and H involved

4) Write down the formula triangle — 

5) You want the angle so cover up C to give — $C = \frac{A}{H}$

6) Put in the numbers — $\cos x = \frac{15}{25} = 0.6$

15 ÷ 25 =

7) Find the inverse. — $\Rightarrow x = \cos^{-1}(0.6) = 53.1301...°$ $= 53.1°$ (1 d.p.)

shift cos 0.6 =

When you're finding an angle you'll have to find the INVERSE at the end. Press SHIFT or 2ndF, followed by sin, cos or tan — your calculator will display $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$

Trigonometry — Common Values

Now that you're in the swing of trigonometry questions it's time to put those calculators away. Sorry.

Learn these Common Trig Values

The tables below contain a load of useful trig values. You might get asked to work out some exact trig answers in your non-calculator exam, so having these in your brain will come in handy.

$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$
$\tan 30° = \frac{1}{\sqrt{3}}$	$\tan 60° = \sqrt{3}$	$\tan 45° = 1$

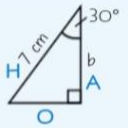
If you're asked for exact answers, don't convert them to decimals at the end.

	$\tan 0° = 0$
$\cos 90° = 0$	$\cos 0° = 1$
$\sin 90° = 1$	$\sin 0° = 0$

Have a look at the examples below — they might help cement a few values into your head.

EXAMPLES:

1. Without using a calculator, find the exact length of side b in the right-angled triangle shown.



1) It's a right-angled triangle so use SOH CAH TOA to pick the correct trig formula to use.

$A = C \times H$

2) Put in the numbers from the diagram in the question.

$b = \cos 30° \times 7$

3) You know the value of cos 30°, so substitute this in.

$b = \frac{\sqrt{3}}{2} \times 7 = \frac{7\sqrt{3}}{2}$ cm

2. Without using a calculator, show that $\cos 60° + \sin 30° = 1$

Put in the right values for cos 60° and sin 30°, then do the sum.

$\cos 60° = \frac{1}{2}$ $\sin 30° = \frac{1}{2}$

$\cos 60° + \sin 30° = \frac{1}{2} + \frac{1}{2} = 1$

How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.
I will also use these at home to complete my own independent learning and revision of these key topics.

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