Knowledge Organiser: Year 9 Maths; Transformations (Part 1)

The Four Transformations

There are four transformations you need to know — translation, rotation, reflection and enlargement.

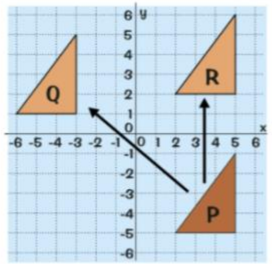
1) Translations

In a translation, the amount the shape moves by is given as a vector (see p.103-104) written $\binom{x}{y}$ — where x is the horizontal movement (i.e. to the right) and y is the vertical movement (i.e. up). If the shape moves left and down, x and y will be negative.

EXAMPLE:
a) Describe the transformation that maps triangle P onto Q.
b) Describe the transformation that maps triangle P onto R.

a) To get from P to Q, you need to move 8 units left and 6 units up, so...
The transformation from P to Q is a translation by the vector $\binom{-8}{6}$.

b) The transformation from P to R is a translation by the vector $\binom{0}{7}$.



2) Rotations

To describe a rotation, you must give 3 details:
1) The angle of rotation (usually 90° or 180°).
2) The direction of rotation (clockwise or anticlockwise).
3) The centre of rotation (often, but not always, the origin).

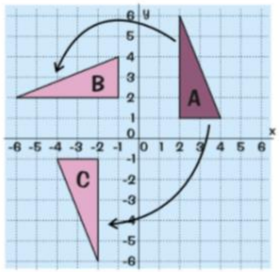
For a rotation of 180°, it doesn't matter whether you go clockwise or anticlockwise.

EXAMPLE:
a) Describe the transformation that maps triangle A onto B.
b) Describe the transformation that maps triangle A onto C.

a) The transformation from A to B is a rotation of 90° anticlockwise about the origin.

b) The transformation from A to C is a rotation of 180° clockwise (or anticlockwise) about the origin.

If it helps, you can use tracing paper to help you find the centre of rotation.

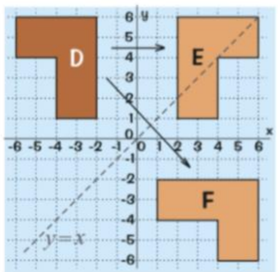


3) Reflections

For a reflection, you must give the equation of the mirror line.

EXAMPLE:
a) Describe the transformation that maps shape D onto shape E.
b) Describe the transformation that maps shape D onto shape F.

a) The transformation from D to E is a reflection in the y-axis.
b) The transformation from D to F is a reflection in the line $y = x$.



The Four Transformations

One more transformation coming up — enlargements. They're the trickiest, but also the most fun (honest).

4) Enlargements

For an enlargement, you must specify:
1) The scale factor.
2) The centre of enlargement.

$$\text{scale factor} = \frac{\text{new length}}{\text{old length}}$$

1) The scale factor for an enlargement tells you how long the sides of the new shape are compared to the old shape. E.g. a scale factor of 3 means you multiply each side length by 3.
2) If you're given the centre of enlargement, then it's vitally important where your new shape is on the grid.

The scale factor tells you the RELATIVE DISTANCE of the old points and new points from the centre of enlargement.

So, a scale factor of 2 means the corners of the enlarged shape are twice as far from the centre of enlargement as the corners of the original shape.

Describing Enlargements

EXAMPLE: Describe the transformation that maps Triangle A onto Triangle B.

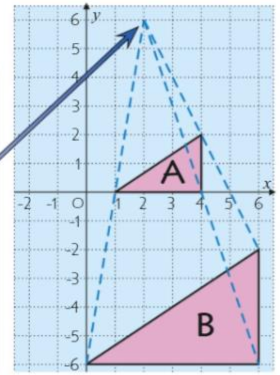
Use the formula to find the scale factor. (Just do this for one pair of sides.)

Old length of triangle base = 3 units
New length of triangle base = 6 units

Scale factor $= \frac{\text{new length}}{\text{old length}} = \frac{6}{3} = 2$

To find the centre of enlargement, draw lines that go through matching corners of both shapes and see where they cross.

So the transformation is an enlargement of scale factor 2, centre (2, 6).

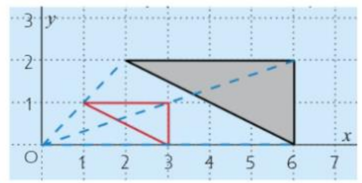


Fractional Scale Factors

1) If the scale factor is bigger than 1 the shape gets bigger.
2) If the scale factor is smaller than 1 (e.g. ½) it gets smaller.

EXAMPLE: Enlarge the shaded shape by a scale factor of $\frac{1}{2}$, about centre O.

1) Draw lines going from the centre to each corner of the original shape. The corners of the new shape will be on these lines.
2) The scale factor is $\frac{1}{2}$, so make each corner of the new shape half as far from O as it is in the original shape.



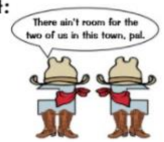
Knowledge Organiser: Year 9 Maths; Transformations (Part 2)

Congruent Shapes

Shapes can be congruent. And I bet you really want to know what that means — I can already picture your eager face. Well, lucky you — I've written a page all about it.

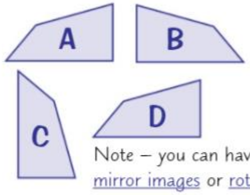
Congruent — Same Shape, Same Size

Congruence is another ridiculous maths word which sounds really complicated when it's not:



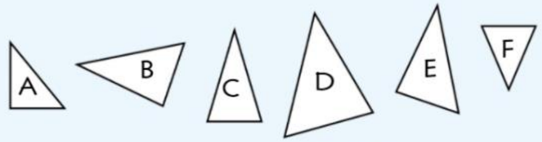
If two shapes are CONGRUENT, they are EXACTLY THE SAME — the SAME SIZE and the SAME SHAPE.

These shapes are all congruent:



Note – you can have mirror images or rotations.

EXAMPLE: Two of the triangles below are congruent. Write down the letters of the congruent triangles.



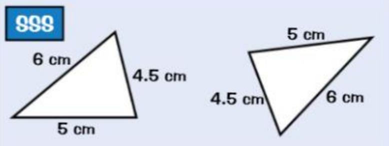
Just pick out the two triangles that are exactly the same — remember that the shape might have been rotated or reflected. By eye, you can see that the congruent triangles are B and E.

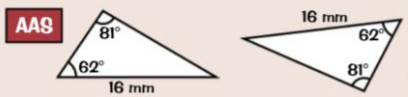
Conditions for Congruent Triangles

Two triangles are congruent if one of the four conditions below holds true:

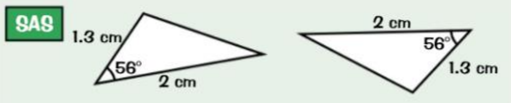
1) SSS three sides are the same
2) AAS two angles and a corresponding side match up
3) SAS two sides and the angle between them match up
4) RHS a right angle, the hypotenuse and one other side all match up

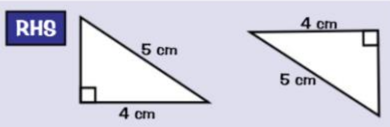
The hypotenuse is the longest side of a right-angled triangle — the one opposite the right angle.





Make sure the sides match up — here, the side is opposite the 81° angle.

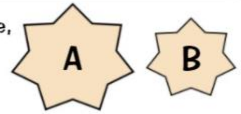




Similar Shapes

Similar shapes are exactly the same shape, but can be different sizes (they can also be rotated or reflected).

SIMILAR — same shape, different size.

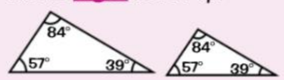


Similar Shapes Have the Same Angles

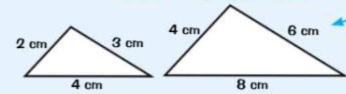
Generally, for two shapes to be similar, all the angles must match and the sides must be proportional. But for triangles, there are three special conditions — if any one of these is true, you know they're similar.

Two triangles are similar if:

1) All the angles match up.

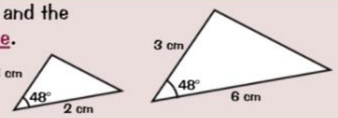


2) All three sides are proportional.



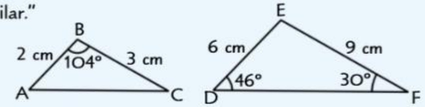
Here, the sides of the bigger triangle are twice as long as the sides of the smaller triangle.

3) Any two sides are proportional and the angle between them is the same.



Watch out — if one of the triangles has been rotated or flipped over, it might look as if they're not similar, but don't be fooled.

EXAMPLE: Tony says, "Triangles ABC and DEF are similar." Is Tony correct? Explain your answer.



Check condition 3 holds — start by finding the missing angle in triangle DEF:

Angle DEF = 180° – 46° – 30° = 104° so angle ABC = angle DEF

Now check that AB and BC are proportional to DE and EF:

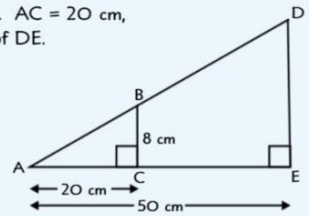
DE ÷ AB = 6 ÷ 2 = 3 and EF ÷ BC = 9 ÷ 3 = 3 so DE and EF are 3 times as long as AB and BC.

Tony is correct — two sides are proportional and the angle between them is the same so the triangles are similar.

Use Similarity to Find Missing Lengths

You might have to use the properties of similar shapes to find missing distances, lengths etc. — you'll need to use scale factors (see p.77) to find the lengths of missing sides.

EXAMPLE: ABC and ADE are similar right-angled triangles. AC = 20 cm, AE = 50 cm and BC = 8 cm. Find the length of DE.



The triangles are similar, so work out the scale factor:

scale factor = $\frac{50}{20}$ = 2.5

Now use the scale factor to work out the length of DE:

DE = 8 × 2.5 = 20 cm

How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

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