Year 13: Half Term 1 – Sequences and Series

Sequences and Series

1

Expansion of $(1+x)^n$ $|x|<1$ $n \in \mathbb{Q}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^3 \ldots\ldots\ldots\ldots + nx^{n-1} + x^n$$

2

Expansion of $(a+b)^n$ $n \in \mathbb{Z}^+$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1\times 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}a^{n-3}b^3 \ldots\ldots\ldots\ldots + nab^{n-1} + b^n$$

3

An inductive definition defines a sequence by giving the first term and a rule to find the next term(s)

$u_{n+1} = f(u_n)$ $u_n = a$

- An **increasing** sequence is one where $u_{n+1} > u_n$ for all n and **decreasing** $u_{n+1} >< u_n$

4

Sigma Notation – sum of

$$\sum_{r=1}^{6}(r^2+1) = (1^2+1) + (2^2+1) + (3^2+1) + (4^2+1) + (5^2+1) + (6^2+1)$$

$= 2 + 5 + 10 + 17 + 26 + 37$

$= 97$

5

Arithmetic sequences and series

- Each term is found by adding a fixed constant (**common difference d)** to the previous term
- The first term is **a** giving the sequence a , a + d, a + 2d, a + 3d...... where $u_n = a + (n-1)d$
- The sum of the first n terms can be found using:

$S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ where *l* is the last term

6

Geometric sequence and series

- Each term is found by multiplying the previous term by a fixed constant (**common ratio r)**
- The first term is **a** giving the sequence a, ar, ar^2, ar^3, ar^4...
- The sum of the first n terms can be found using

$S_n = \frac{a(1-r^n)}{1-r}$ or $S_n = \frac{a(r^n-1)}{r-1}$ $S_\infty = \frac{a}{1-r}$ |r| < 1

Differentiation

1

- The gradient is denoted by $\frac{dy}{dx}$ if y is given as a function of x
- The gradient is denoted by f'(x) is the function is given as f(x)

2

$y = x^n \quad \frac{dy}{dx} = nx^{n-1}$ $\qquad$ $y = ax^n \quad \frac{dy}{dx} = nax^{n-1}$

$y = a \quad \frac{dy}{dx} = 0$ $\qquad$ $y = e^{kx} \quad \frac{dy}{dx} = ke^x$

$y = lnx \quad \frac{dy}{dx} = \frac{1}{x}$ $\qquad$ $y = a^{kx} \quad \frac{dy}{dx} = (klna)a^{kx}$

$y = \cos kx \quad \frac{dy}{dx} = -k\sin kx$ $\qquad$ $y = \tan kx \quad \frac{dy}{dx} = ksec^2kx$

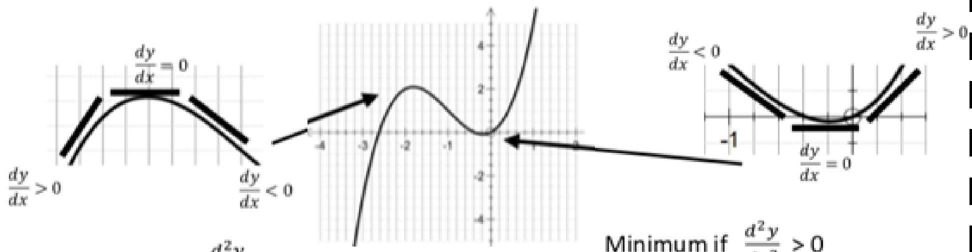
4

Stationary Points

- The points where $\frac{dy}{dx} = 0$ are stationary points of a graph
- The nature of the turning points can be found by:

Maximum point

Minimum Point



Maximum if $\frac{d^2y}{dx^2} < 0$

Minimum if $\frac{d^2y}{dx^2} > 0$

3

Methods of differentiation

CHAIN RULE for differentiating **y = fg(x)** y = f(u) where u = g(x) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

PRODUCT RULE for differentiating **y = f(x)g(x)** $\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$

QUOTIENT RULE for differentiating **y =** $\frac{f(x)}{g(x)}$ $\frac{dy}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

PARAMETRIC EQUATIONS y = f(t) x = g(t) $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

5

- **Points of inflection** occur when $\frac{d^2y}{dx^2} = 0$
- **Convex curve:** $\frac{d^2y}{dx^2} > 0$ for all values of x in the convex section of the curve
- **Concave curve:** $\frac{d^2y}{dx^2} < 0$ for all values of x in the concave section of the curve

How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

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