Year 13: Half Term 1 – Functions

Functions

1

A function is a rule which generates exactly ONE OUTPUT for EVERY INPUT

- **DOMAIN** – defines the set of the values that can be 'put into' the function $f(x) = \sqrt{x}$ domain $x \geq 0$
- **RANGE** – defines the set of values 'output' by the function – make sure it is defined in terms of f(x) and not x

  $f : x \mapsto x^2 \quad x \in \mathbb{R}$ means an input a is converted to a^2 where the input 'a' can be any real number

  $Range\ f(x) \geq 0$

- **INVERSE FUNCTION** denoted by $f^{-1}(x)$

  The domain of $f^{-1}(x)$ is the range of f(x)

  The range of $f^{-1}(x)$ is the domain of f(x)

  Using the same scale on the x and y axis the graphs of a function and it's inverse have **reflection symmetry** in the line **y = x**

$f(x) = \frac{3}{x+2}$ find $f^{-1}(x)$

$y = \frac{3}{x+2}$

$x = \frac{3}{y} - 2$

$f^{-1}(x) = \frac{3}{x} - 2$

- **COMPOSITE FUNCTIONS**

  The function **gf**(x) is a composite function which tells you 'to do' **f** first and then use the output in **g**

f(x) = 4x g(x) = $x^2 - 1$

fg(x) = $4(x^2 - 1)$
= $4x^2 - 4$

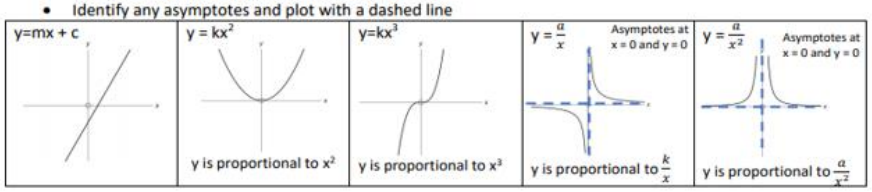
gf(x) = $(4x)^2 - 1$
= $16x^2 - 1$

2

GRAPHS OF FUNCTIONS

Sketching Graphs

- Identify where the graph crossed the y-axis (x = 0)
- Identify where the graph crossed the x-axis (y = 0)
- Identify any asymptotes and plot with a dashed line



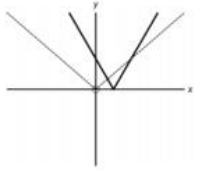
Modulus Graphs

- |x| is the 'modulus of x' or the absolute value |2|=2 |-2|= 2
- To sketch the graph of y = |f(x)| sketch y = f(x) and take any part of the graph which is below the x-axis and reflect it in the x-axis

Solve |2x - 4|<|x|

2x - 4 = x 2x – 4 = -x
x = 4 3x = 4
$x = \frac{4}{3}$

$\frac{4}{3} < x < 4$



3

TRANSFORMING GRAPHS

Translation

To find the equation of a graph after a translation of $\begin{bmatrix} a \\ b \end{bmatrix}$ replace x by (x – a) and y by (y – b)

y = f(x -a) + b

The graph of $y = x^2 - 1$ is translated by $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

Find the equation of the resulting graph.

$(y + 2) = (x - 3)^2 - 1$

$y = x^2 - 6x + 6$

Reflection

Reflection in the x-axis replace y with -y y = -f(x)

Reflection in the y-axis replace x with -x y = f(-x)

Stretch

Stretch in the y-direction by scale factor a y = af(x)

Stretch on the x-direction by scale factor $\frac{1}{a}$ y = f(ax)

Combining Transformations

Take care with the order in which the transformations are carried out.

The graph of $y = x^2$ **is translated by** $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ and then **reflected in the y axis**. Find the equation of the resulting graph

Translation $y = (x - 3)^2$
$= x^2 - 6x + 9$

Reflection $y = (-x)^2 - 6(-x) + 9$
$= \mathbf{x^2 + 6x + 9}$

The graph of $y = x^2$ is **reflected in the y axis** and then **translated by** $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$. Find the equation of the resulting graph

Reflection $y = (-x)^2$
$= x^2$

Translation $y = (x - 3)^2$
$= \mathbf{x^2 - 6x + 9}$

How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

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GLUE HERE

Trigonometry

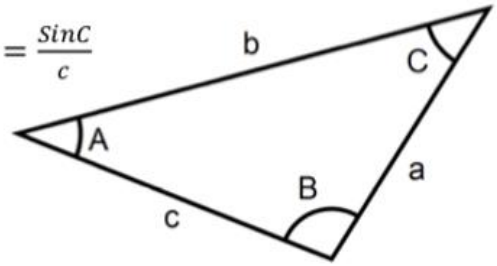
1

MAKE SURE YOU KNOW AND CAN USE THE FOLLOWING FROM GCSE

$Area = \frac{1}{2}absinC$

$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ or $\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$

$a^2 = b^2 + c^2 - 2bcCosA$



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	1
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	---	0

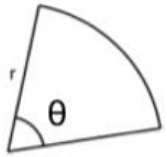
2

Radians 2π radians = 360° π radians = 180°

- You MUST work in radians if you are integrating or differentiating trig functions
- For an angle at the centre of a sector of θ **radians**

Arc Length = rθ

Area of the sector = ½r²θ



3

Small angle approximations (θ in radians)

$sin\theta \approx \theta$ $cos\theta \approx 1 - \frac{\theta^2}{2}$ $tan\theta \approx \theta$

When θ is small show that $\frac{cos\theta}{sin\theta}$ can be written as $\frac{2-\theta^2}{2\theta}$

$\left(1 - \frac{\theta^2}{2}\right) \div \theta$

$= \frac{2-\theta^2}{2} \div \theta$

$= \frac{2-\theta^2}{2\theta}$

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