Year 12: Half Term 1 – Surds and Indices (Part 1)

Surds

1

- A root such as $\sqrt{3}$ that cannot be written exactly as a fraction is IRRATIONAL
- An expression that involves irrational roots is in SURD FORM e.g. $2\sqrt{3}$
- $3+\sqrt{2}$ and $3-\sqrt{2}$ are CONJUGATE/COMPLEMENTARY surds – needed to rationalise the denominator

2

SIMPLIFYING $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Simplify $\sqrt{75} - \sqrt{12}$

$= \sqrt{5 \times 5 \times 3} - \sqrt{2 \times 2 \times 3}$

$= 5\sqrt{3} - 2\sqrt{3}$

$= 3\sqrt{3}$

3

RATIONALISING THE DENOMINATOR (removing the surd in the denominator)

$a+\sqrt{b}$ and $a-\sqrt{b}$ are CONJUGATE/COMPLEMENTARY surds – the product is always a rational number

Rationalise the denominator $\frac{2}{2-\sqrt{3}}$

$= \frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{4+2\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3}$

$= 4+2\sqrt{3}$

Multiply the numerator and denominator by the conjugate of the denominator

Indices

1

Rules to learn

$x^a \times x^b = x^{a+b}$ $\qquad x^{-a} = \frac{1}{x^a}$

$x^a \div x^b = x^{a-b}$ $\qquad x^{\frac{1}{n}} = \sqrt[n]{x}$

$(x^a)^b = x^{ab}$ $\qquad x^{\frac{m}{n}} = \sqrt[n]{x^m}$

2

Solve the equation $25^x = (5^2)^x$

$3^{2x} \times 25^x = 15$

$(3 \times 5)^{2x} = (15)^1$

$2x = 1$

$x = \frac{1}{2}$

3

$(x-y)^{\frac{3}{2}} = (x-y)^{\frac{1}{2}}(x-y)$

Simplify

$2x(x-y)^{\frac{3}{2}} + 3(x-y)^{\frac{1}{2}}$

$(x-y)^{\frac{1}{2}}(2x(x-y)+3)$

$(x-y)^{\frac{1}{2}}(2x^2-2xy+3)$

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Year 12: Half Term 1 – Equations and Inequalities (Part 2)

Simultaneous Equation

1 Solving by elimination

$3x - 2y = 19$ × 3 $9x - 6y = 57$

$2x - 3y = 21$ × 2 $4x - 6y = 42$

$5x - 0y = 15$ $x = 3$ $(9 - 2y = 19)$ $y = -5$

3

If when solving a pair of simultaneous equations, you arrive with a quadratic equation to solve, this can be used to determine the relationship between the graphs of the original equations

Using the discriminant

$b^2 - 4ac > 0$ the graphs intersect at 2 distinct points

$b^2 - 4ac = 0$ the graphs intersect at 1 point (tangent)

$b^2 - 4a < 0$ the graphs do not intersect

2 Solving by substitution

$x + y = 1$ rearranges to $y = 1 - x$

$x^2 + y^2 = 25$

$x^2 + (1 - x)^2 = 25$

$x^2 + 1 - 2x + x^2 - 25 = 0$

$2x^2 - 2x - 24 = 0$

$2(x^2 - x - 12) = 0$

$2(x - 4)(x + 3) = 0$ $x = 4$ $x = -3$

$y = -3$ $y = 4$

Inequalities

1

Linear Inequality

This can be solved like a linear equation except that

Multiplying or Dividing by a negative value reverses the inequality

2

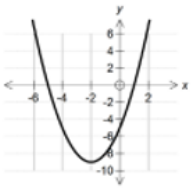
Quadratic Inequality – always a good idea to sketch the graph!

Solve $x^2 + 4x - 5 < 0$

$x^2 + 4x - 5 = 0$

$(x - 1)(x + 5) = 0$

$x = 1$ $x = -5$



$x^2 + 4x - 5 < 0$

$-5 < x < 1$

which can be written as

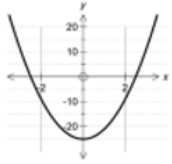
$\{x : x > -5\} \cap \{x : x < 1\}$

Solve $4x^2 - 25 \geq 0$

$4x^2 - 25 = 0$

$(2x - 5)(2x + 5) = 0$

$x = \frac{5}{2}$ $x = -\frac{5}{2}$



$4x^2 - 25 \geq 0$

$x \leq -\frac{5}{2}$ or $x \geq \frac{5}{2}$

which can be written as

$\{x : x \leq -\frac{5}{2}\} \cup \{x : x \geq \frac{5}{2}\}$

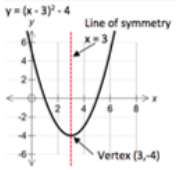
Quadratic Equation and Graphs

1 Factorising identifying the roots of the equation $ax^2 + bc + c = 0$

- Look out for the difference of 2 squares $x^2 - a^2 = (x - a)(x + a)$
- Look out for the perfect square $x^2 + 2ax + a^2 = (x + a)^2$ or $x^2 - 2ax + a^2 = (x - a)^2$
- Look out for equations which can be transformed into quadratic equations

2 Completing the square - Identifying the vertex and line of symmetry

In completed square form $y = (x + a)^2 + b$



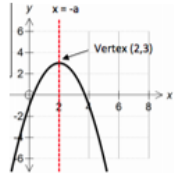
Sketch the graph of

$y = 4x - x^2 - 1$

$y = -(x^2 - 4x) - 1$

$y = -((x - 2)^2 - 4) - 1$

$y = -(x-2)^2 + 3$



3

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ for solving $ax^2 + bx + c = 0$

The **DISCRIMINANT $b^2 - 4ac$** can be used to identify the number of solutions

$b^2 - 4ac > 0$ there are 2 real and distinct roots (the graphs crosses the x- axis in 2 places)

$b^2 - 4ac = 0$ the is a single repeated root (the x-axis is a tangent to the graph)

$b^2 - 4ac < 0$ there are no 2 real roots (the graph does not touch or cross the x-axis)

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