

# Integration

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Integration is the reverse of differentiation

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$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (c \text{ is the constant of integration})$$

$$\int e^{kx} dx = \frac{1}{k} e^{kx} + c$$

$$\int \sin kx dx = -\frac{1}{k} \cos kx + c$$

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int \cos kx dx = \frac{1}{k} \sin kx + c$$

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## INTEGRATION BY SUBSTITUTION

Transforming a complex integral into a simpler integral using 'u = ' and integrating with respect to u

$$\begin{aligned} & \int x\sqrt{1-x^2} dx \\ \text{Let } u = 1 - x^2 & \quad \frac{du}{dx} = -2x \quad \text{so } dx = \frac{du}{-2x} \\ \int x\sqrt{1-x^2} dx &= \int x\sqrt{u} \frac{du}{-2x} \\ &= -\frac{1}{2} \int u^{\frac{1}{2}} du \\ &= -\frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + c \\ &= -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + c \end{aligned}$$

If it is a definite integral it is often easier to calculate the limits in terms of u and substitute these in after integrating

Look for integrals of the form

$$\int e^{ax+b} dx \quad \int \cos(ax+b) dx \quad \int \frac{1}{ax+b} dx$$

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## Parametric Integration

To find the area under a curve defined parametrically use

$$\text{area} \int y \frac{dx}{dy} dt$$

Remember that the limits of the integral must be in terms of t

The area under a graph can be approximated using rectangle of height y and width dx. The limit as the number of rectangles increases is equal to definite integral

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n y_i \partial x = \int_a^b y dx$$

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## INTEGRATION BY PARTS

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Take care in defining u and  $\frac{dv}{dx}$

$$\int x e^{2x} dx \quad u = x \quad \frac{dv}{dx} = e^{2x}$$

$$\int x \ln x dx \quad u = \ln x \quad \frac{dv}{dx} = x$$



## How do we use Knowledge Organisers in Mathematics?

### How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

### How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

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