Year 12: Half Term 5 – Differentiation

Differentiation

1

- The gradient is denoted by $\frac{dy}{dx}$ if y is given as a function of x
- The gradient is denoted by f'(x) is the function is given as f(x)

$y = x^n \quad \frac{dy}{dx} = nx^{n-1}$ $\qquad y = ax^n \quad \frac{dy}{dx} = nax^{n-1}$ $\qquad y = a \quad \frac{dy}{dx} = 0$

Using Differentiation

2

Tangents and Normals

The gradient of a curve at a given point = gradient of the tangent to the curve at that point
The gradient of the **normal** is perpendicular to the gradient of the tangent that point

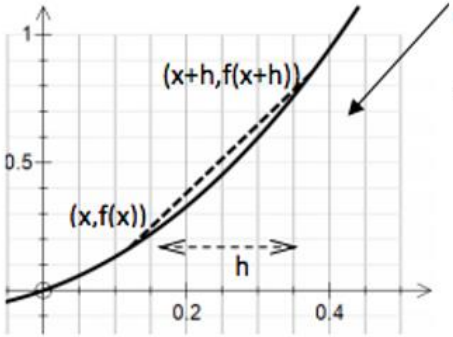
3

Stationary (Turning) Points

- The points where $\frac{dy}{dx} = 0$ are stationary points (turning points/points of inflection) of a graph
- The nature of the turning points can be found by:

Differentiation from first principles

4



As h approaches zero the gradient of the chord gets closer to being the gradient of the tangent at the point

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Find from first principles the derivative of $x^3 - 2x + 3$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{(x+h)^3-2(x+h)+3-(x^3-2x+3)}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{x^3+3x^2h+3xh^2+h^3-2x-2h+3-x^3+2x-3}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{3x^2h+3xh^2+h^3-2h}{h}\right)$$
$$= \lim_{h\to 0}(3x^2+3xh+h^2-2)$$
$$= 3x^2 - 2$$

Calculating the gradient close to the point

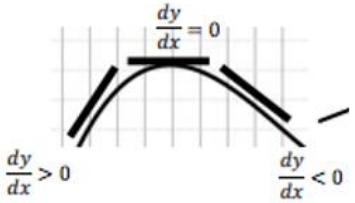
5

Differentiating (again) to find $\frac{d^2y}{dx^2}$ or f''(x)

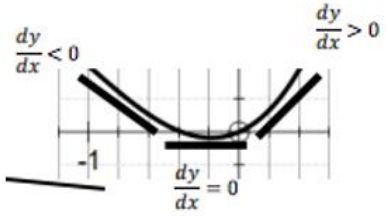
Maximum if $\frac{d^2y}{dx^2} < 0$

Minimum if $\frac{d^2y}{dx^2} > 0$

Maximum point



Minimum Point



How do we use Knowledge Organisers in Mathematics?

How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.
I will also use these at home to complete my own independent learning and revision of these key topics.

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