Knowledge Organiser: Year 10 Foundation (Spring)

1

Pythagoras' Theorem

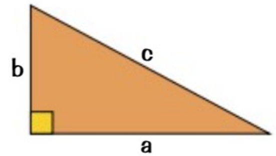
Once upon a time there lived a clever chap called Pythagoras. He came up with a clever theorem...

Pythagoras' Theorem is Used on Right-Angled Triangles

Pythagoras' theorem only works for RIGHT-ANGLED TRIANGLES.
It uses two sides to find the third side.

The formula for Pythagoras' theorem is: $a^2 + b^2 = c^2$

short sides — long side



The trouble is, the formula can be quite difficult to use. Instead, it's a lot better to just remember these three simple steps, which work every time:

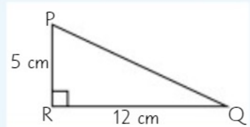
1) SQUARE THEM — SQUARE THE TWO NUMBERS that you are given, (use the x^2 button if you've got your calculator.)

2) ADD or SUBTRACT — To find the longest side, ADD the two squared numbers. $a^2 + b^2 = c^2$
To find a shorter side, SUBTRACT the smaller from the larger. $c^2 - b^2 = a^2$

3) SQUARE ROOT — Once you've got your answer, take the SQUARE ROOT (use the $\sqrt{\ }$ button on your calculator). $c = \sqrt{a^2 + b^2}$ $a = \sqrt{c^2 - b^2}$

EXAMPLES:

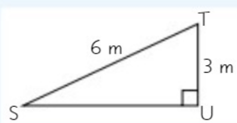
1. Find the length of side PQ in this triangle.



1) Square them: $a^2 = 5^2 = 25$, $b^2 = 12^2 = 144$
2) You want to find the longest side, so ADD: $a^2 + b^2 = c^2$
$25 + 144 = 169$
3) Square root: $c = \sqrt{169} = 13$ cm

Always check the answer's sensible — 13 cm is longer than the other two sides, but not too much longer, so it seems OK.

2. Find the length of SU to 1 decimal place.



1) Square them: $b^2 = 3^2 = 9$, $c^2 = 6^2 = 36$
2) You want to find a shorter side, so SUBTRACT: $c^2 - b^2 = a^2$
$36 - 9 = 27$
3) Square root: $a = \sqrt{27} = 5.196...$
$= 5.2$ m (to 1 d.p.)

Check the answer is sensible — yes, it's a bit shorter than the longest side.

2

LCM and HCF

As if the previous page wasn't enough excitement, here's some more factors and multiples fun...

LCM — 'Least Common Multiple'

The SMALLEST number that will DIVIDE BY ALL the numbers in question.

If you're given two numbers and asked to find their LCM, just LIST the MULTIPLES of BOTH numbers and find the SMALLEST one that's in BOTH lists.

So, to find the LCM of 12 and 15, list their multiples (multiples of 12 = 12, 24, 36, 48, 60, 72, ... and multiples of 15 = 15, 30, 45, 60, 75, ...) and find the smallest one that's in both lists — so LCM = 60.

However, if you already know the prime factors of the numbers, you can use this method instead:

1) List all the PRIME FACTORS that appear in EITHER number.
2) If a factor appears MORE THAN ONCE in one of the numbers, list it THAT MANY TIMES.
3) MULTIPLY these together to give the LCM.

EXAMPLE: $18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$. Find the LCM of 18 and 30.

$18 = 2 \times 3 \times 3$ $30 = 2 \times 3 \times 5$

So the prime factors that appear in either number are: 2, 3, 3, 5 — List 3 twice as it appears twice in 18.

LCM $= 2 \times 3 \times 3 \times 5 = 90$

HCF — 'Highest Common Factor'

The BIGGEST number that will DIVIDE INTO ALL the numbers in question.

If you're given two numbers and asked to find their HCF, just LIST the FACTORS of BOTH numbers and find the BIGGEST one that's in BOTH lists.

Take care listing the factors — make sure you use the proper method (as shown on the previous page).

So, to find the HCF of 36 and 54, list their factors (factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18 and 36 and factors of 54 = 1, 2, 3, 6, 9, 18, 27 and 54) and find the biggest one that's in both lists — so HCF = 18.

Again, there's a different method you can use if you already know the prime factors of the numbers:

1) List all the PRIME FACTORS that appear in BOTH numbers.
2) MULTIPLY these together to find the HCF.

EXAMPLE: $180 = 2^2 \times 3^2 \times 5$ and $84 = 2^2 \times 3 \times 7$. Use this to find the HCF of 180 and 84.

$180 = 2 \times 2 \times 3 \times 3 \times 5$ $84 = 2 \times 2 \times 3 \times 7$

2, 2 and 3 are prime factors of both numbers, so HCF $= 2 \times 2 \times 3 = 12$

4

Six Steps for Easy Simultaneous Equations

Grade 5

EXAMPLE: Solve the simultaneous equations $2x = 6 - 4y$ and $-3 - 3y = 4x$

1. Rearrange both equations into the form $ax + by = c$, and label the two equations ① and ②.

a, b and c are numbers (which can be negative)

$2x + 4y = 6$ — ①
$4x + 3y = -3$ — ②

2. Match up the numbers in front (the 'coefficients') of either the x's or y's in both equations. You may need to multiply one or both equations by a suitable number. Relabel them ③ and ④.

① × 2: $4x + 8y = 12$ — ③
$4x + 3y = -3$ — ④

3. Add or subtract the two equations to eliminate the terms with the same coefficient.

③ − ④ $0x + 5y = 15$

If the coefficients have the same sign (both +ve or both −ve) then subtract. If the coefficients have opposite signs (one +ve and one −ve) then add.

4. Solve the resulting equation.

$5y = 15 \Rightarrow y = 3$

5. Substitute the value you've found back into equation ① and solve it.

Sub $y = 3$ into ①: $2x + (4 \times 3) = 6 \Rightarrow 2x + 12 = 6 \Rightarrow 2x = -6 \Rightarrow x = -3$

6. Substitute both these values into equation ② to make sure it works. If it doesn't then you've done something wrong and you'll have to do it all again.

Sub x and y into ②: $(4 \times -3) + (3 \times 3) = -12 + 9 = -3$, which is right, so it's worked.

So the solutions are: $x = -3$, $y = 3$

Powers

3

Powers are a very Useful Shorthand

Grade 4

1. Powers are 'numbers multiplied by themselves so many times': $2\times2\times2\times2\times2\times2\times2 = 2^7$ ('two to the power 7')
2. The powers of ten are really easy — the power tells you the number of zeros:
   $10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^6 = 1000000$ (to the power of 6 — 6 zeros)
3. Use the $x^\blacksquare$ or y^x button on your calculator to find powers, e.g. press 3 · 7 $x^\blacksquare$ 3 = to get $3.7^3 = 50.653$.
4. Anything to the power 1 is just itself, e.g. $4^1 = 4$.
5. 1 to any power is still 1, e.g. $1^{457} = 1$.
6. Anything to the power 0 is just 1, e.g. $5^0 = 1$, $67^0 = 1$, $x^0 = 1$.

Four Easy Rules:

Grade 4

1) When MULTIPLYING, you ADD THE POWERS. e.g. $3^4 \times 3^6 = 3^{4+6} = 3^{10}$
2) When DIVIDING, you SUBTRACT THE POWERS. e.g. $c^4 \div c^2 = c^{4-2} = c^2$
3) When RAISING one power to another, you MULTIPLY THE POWERS. e.g. $(3^2)^4 = 3^{2\times4} = 3^8$
4) FRACTIONS — Apply the power to both TOP and BOTTOM. e.g. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

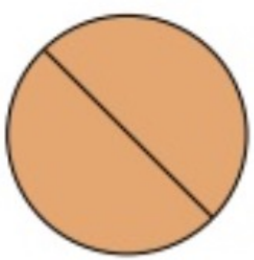
Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

EXAMPLE: $a = 5^9$ and $b = 5^4 \times 5^2$. What is the value of $\frac{a}{b}$?

1) Work out b — add the powers: $b = 5^4 \times 5^2 = 5^{4+2} = 5^6$
2) Divide a by b — subtract the powers: $\frac{a}{b} = 5^9 \div 5^6 = 5^{9-6} = 5^3 = 125$

Area and Circumference of Circles

Grade 3



Area of circle $= \pi \times (\text{radius})^2$

Remember that the radius is half the diameter.

$$A = \pi r^2$$

For these formulas, use the π button on your calculator. For non-calculator questions, use $\pi \approx 3.142$.

5

Circumference $= \pi \times \text{diameter}$
$= 2 \times \pi \times \text{radius}$

$$C = \pi D = 2\pi r$$

Formulas and Equations from Words

Sometimes, you might be asked to use an expression to solve an equation.

EXAMPLE: A zoo has x zebras and four times as many lemurs. The difference between the number of zebras and the number of lemurs is 45. How many zebras does the zoo have?

The zoo has x zebras and $4 \times x = 4x$ lemurs.

The difference is $4x - x = 3x$, so $3x = 45$, which means $x = 15$. — Once you've formed the equation, you need to solve it to find the value of x.

So the zoo has 15 zebras.

EXAMPLE: Will, Naveed and Camille give some books to charity. Naveed gives 6 more books than Will, and Camille gives 7 more books than Naveed. Between them, they give away 46 books. How many books did they give each?

Let the number of books Will gives be x.
Then Naveed gives $x + 6$ books
and Camille gives $(x + 6) + 7 = x + 13$ books
So in total they give $x + x + 6 + x + 13 = 3x + 19$ books

So $3x + 19 = 46$ — You're told this in the question.
$3x = 27$
$x = 9$

So Will gives 9 books,
Naveed gives $9 + 6 = 15$ books and
Camille gives $15 + 7 = 22$ books.

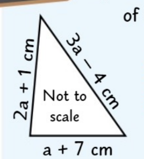
Use Shape Properties to Find Formulas and Equations

GRADE 4

In some questions, you'll need to use what you know about shapes (e.g. side lengths or areas) to come up with a formula or an equation to solve.

EXAMPLE:

a) Write a formula for P, the perimeter of the triangle below, in terms of a.



Form an expression for the perimeter:
P = (a + 7) + (2a + 1) + (3a – 4)
P = 6a + 4 cm

b) If the triangle has a perimeter of 58 cm, find the value of a.

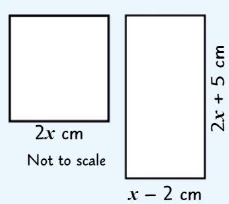
P = 58, so set your formula equal to 58 and solve to find a:
6a + 4 = 58
6a = 54
a = 9

Compare Dimensions of Two Shapes to Find Equations

GRADE 5

You might get a question that involves two shapes with related areas or perimeters — you'll have to use this fact to find side lengths or missing values.

EXAMPLE: The perimeter of the rectangle is the same as the perimeter of the square. Find the value of x.



Perimeter of square $= 2x + 2x + 2x + 2x = 8x$ cm
Perimeter of rectangle $= (2x + 5) + (x - 2) + (2x + 5) + (x - 2)$
$= 6x + 6$ cm

Set the perimeter of the rectangle equal to the perimeter of the square and solve:
$8x = 6x + 6$
$2x = 6$
$x = 3$

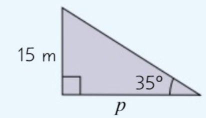
Trigonometry — Examples

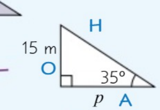
Here are some lovely examples using the method from p.96 to help you through the trials of trig.

Examples:

GRADE 6

1 Find the length of p in the triangle shown to 3 s.f.





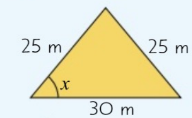
1) Label the sides
2) Write down — SOH CAH TOA
3) O and A involved
4) Write down the formula triangle

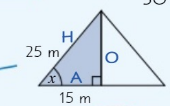
5) You want A so cover it up to give — $A = \frac{O}{T}$
6) Put in the numbers — $p = \frac{15}{\tan 35°} = 21.422...$
$= 21.4$ m (3 s.f.)

Is it sensible? Yes, it's a bit bigger than 15, as the diagram suggests.

2 Find the angle x in this triangle to 1 d.p.

It's an isosceles triangle so split it down the middle to get a right-angled triangle.





1) Label the sides
2) Write down — SOH CAH TOA
3) A and H involved
4) Write down the formula triangle

5) You want the angle so cover up C to give — $C = \frac{A}{H}$
6) Put in the numbers — $\cos x = \frac{15}{25} = 0.6$
7) Find the inverse — $\Rightarrow x = \cos^{-1}(0.6) = 53.1301...°$
$= 53.1°$ (1 d.p.)

Is it sensible? Yes, the angle looks about 50°.

Multiplying Out Brackets

I usually use brackets to make witty comments (I'm very witty), but in algebra they're useful for simplifying things. First of all, you need to know how to expand brackets (multiply them out).

Single Brackets

GRADE 3

The main thing to remember when multiplying out brackets is that the thing outside the bracket multiplies each separate term inside the bracket.

EXAMPLE: Expand the following:

a) $4a(3b - 2c)$
$= (4a \times 3b) + (4a \times -2c)$
$= 12ab - 8ac$

b) $-4(3p^2 - 7q^3)$
$= (-4 \times 3p^2) + (-4 \times -7q^3)$
$= -12p^2 + 28q^3$

Note: both signs have been reversed.

Compound Growth and Decay

6

One more sneaky % type for you... Unlike simple interest, in compound interest the amount added on (or taken away) changes each time — it's a percentage of the new amount, rather than the original amount.

The Formula (Grade 5)

This topic is simple if you LEARN THIS FORMULA. If you don't, it's pretty well impossible:

$$N = N_0 \times (\text{multiplier})^n$$

- Amount after n days/hours/years: N
- Initial amount: N_0
- Number of days/hours/years: n
- Percentage change multiplier
  E.g. 5% increase is 1.05 (= 1 + 0.05)
  26% decrease is 0.74 (= 1 – 0.26)

3 Examples to show you how EASY it is: (Grade 5)

Compound interest is a popular context for these questions — it means the interest is added on each time, and the next lot of interest is calculated using the new total rather than the original amount.

EXAMPLE: Daniel invests £1000 in a savings account which pays 8% compound interest per annum. How much will there be after 6 years?

'Per annum' just means 'each year'.

Use the formula: $\text{Amount} = \underbrace{1000}_{\text{initial amount}}(\underbrace{1.08}_{\text{8\% increase}})^{\overbrace{6}^{\text{6 years}}} = £1586.87$

Depreciation questions are about things (e.g. cars) which decrease in value over time.

EXAMPLE: Susan has just bought a car for £6500.

a) If the car depreciates by 9% each year, how much will it be worth in 3 years' time?

Use the formula: Value = $6500(0.91)^3$ = £4898.21

b) How many complete years will it be before the car is worth less than £3000?

Use the formula again but this time you know don't know n.

Value = $6500(0.91)^n$

Use trial and error to find how many years it will be before the value drops below £3000.

If $n = 8$, $6500(0.91)^8 = 3056.6414...$
$n = 9$, $6500(0.91)^9 = 2781.5437...$

It will be 9 years before the car is worth less than £3000.

Percentages

7

Type 6 — Finding the Original Value (Grade 4)

This is the type that most people get wrong — but only because they don't recognise it as this type, and don't apply this simple method:

1) Write the amount in the question as a percentage of the original value.
2) Divide to find 1% of the original value.
3) Multiply by 100 to give the original value (= 100%).

EXAMPLE: A house increases in value by 10% to £165 000. Find what it was worth before the rise.

Note: The new, not the original value is given.

1) An increase of 10% means £165 000 represents 110% of the original value.
2) Divide by 110 to find 1% of the original value.
3) Then multiply by 100.

$$\begin{aligned} £165\,000 &= 110\% \\ \div 110 \quad £1500 &= 1\% \\ \times 100 \quad £150\,000 &= 100\% \end{aligned}$$

If it was a decrease of 10%, then you'd put '£165 000 = 90%' and divide by 90 instead of 110.

So the original value was £150 000

Always set them out exactly like this example. The trickiest bit is deciding the top % figure on the right-hand side — the 2nd and 3rd rows are always 1% and 100%.

Prime Numbers: (Grade 3)

2 3 5 7 11 13 17 19 23 29 31 37 41 43...

A prime number is a number which doesn't divide by anything, apart from itself and 1 — i.e. its only factors are itself and 1. (The only exception is 1, which is NOT a prime number.)

8

Doing the 'Table of Values' (Grade 3)

EXAMPLE: Draw the graph of $y = 2x - 3$ for values of x from –2 to 4.

1. Choose 3 easy x-values for your table:
Use x-values from the grid you're given.
Avoid negative ones if you can.

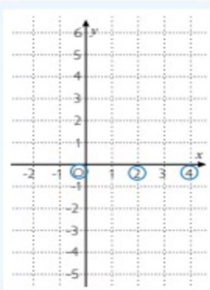
x	0	2	4
y			

2. Find the y-values by putting each x-value into the equation:

x	0	2	4
y	–3	1	5

When $x = 0$,
$y = 2x - 3$
$= (2 \times 0) - 3 = -3$

When $x = 4$,
$y = 2x - 3$
$= (2 \times 4) - 3 = 5$



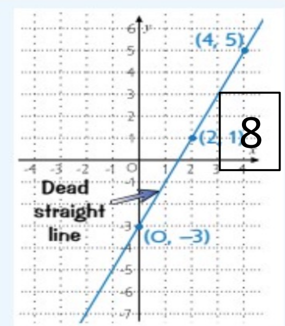
Plotting the Points and Drawing the Graph (Grade 3)

EXAMPLE: ...continued from above.

3. PLOT EACH PAIR of x- and y- values from your table.
The table gives the coordinates (0, –3), (2, 1) and (4, 5).

Now draw a STRAIGHT LINE through your points.

If one point looks a bit wacky, check 2 things:
– the y-values you worked out in the table
– that you've plotted the points properly.



The More Awkward Cases:

Grade 3

1) If the ratio contains decimals or fractions — multiply

For fractions, multiply by a number that gets rid of both denominators.

EXAMPLE: Simplify the ratio 2.4:3.6 as far as possible.

1) Multiply both sides by 10 to get rid of the decimal parts.
2) Now divide to reduce the ratio to its simplest form.

$$2.4:3.6 \xrightarrow{\times 10} = 24:36 \xrightarrow{\div 12} = 2:3$$

2) If the ratio has mixed units — convert to the smaller unit

EXAMPLE: Reduce the ratio 24 mm:7.2 cm to its simplest form.

1) Convert 7.2 cm to millimetres.
2) Simplify the resulting ratio. Once the units on both sides are the same, get rid of them for the final answer.

24 mm:7.2 cm
= 24 mm:72 mm
$\div 24$ = 1:3 $\div 24$

3) To get to the form 1 : n or n : 1 — just divide

EXAMPLE: Reduce 3:56 to the form 1:n.

This form is often the most useful, since it shows the ratio very clearly.

Divide both sides by 3:

$$3:56 \xrightarrow{\div 3} = 1:\frac{56}{3} = 1:18\frac{2}{3} \text{ (or } 1:18.\dot{6}\text{)}$$

Estimating Calculations

Grade 4

Have a look at the previous page to remind yourself how to round to 1 s.f.

1. Round everything off to 1 significant figure.
2. Then work out the answer using these nice easy numbers.
3. Show all your working or you won't get the marks.

EXAMPLES:

1. Estimate the value of 42.6 × 12.1.

$\approx$ means 'approximately equal to'.

1) Round each number to 1 s.f.
2) Do the calculation with the rounded numbers.

$42.6 \times 12.1 \approx 40 \times 10$
$= 400$

You might have to say if it's an underestimate or an overestimate. Here, you rounded both numbers down, so it's an underestimate.

2. Estimate the value of $\frac{\sqrt{6242 \div 57}}{9.8 - 4.7}$.

Don't be put off by the square root, just round each number to 1 s.f. and do the calculation.

$$\frac{\sqrt{6242 \div 57}}{9.8 - 4.7} \approx \frac{\sqrt{6000 \div 60}}{10 - 5} = \frac{\sqrt{100}}{5} = \frac{10}{5} = 2$$

3. Jo has a cake-making business. She spent £984.69 on flour last year. A bag of flour costs £1.89, and she makes an average of 5 cakes from each bag of flour. Work out an estimate of how many cakes she made last year.

Don't panic if you get a 'real-life' estimating question — just round everything to 1 s.f. as before.

1) Estimate number of bags of flour — round numbers to 1 s.f.

Number of bags of flour $= \frac{984.69}{1.89} \approx \frac{1000}{2} = 500$

2) Multiply to find the number of cakes.

Number of cakes $\approx 500 \times 5 = 2500$

Part : Whole Ratios

Grade 3

You might come across a ratio where the LHS is included in the RHS — these are called part : whole ratios.

EXAMPLE: Mrs Miggins owns tabby cats and ginger cats. The ratio of tabby cats to the total number of cats is 3:5.

a) What fraction of Mrs Miggins' cats are tabby cats?

The ratio tells you that for every 5 cats, 3 are tabby cats. $\frac{\text{part}}{\text{whole}} = \frac{3}{5}$

b) What is the ratio of tabby cats to ginger cats?

3 in every 5 cats are tabby, so 2 in every 5 are ginger. $5 - 3 = 2$

For every 3 tabby cats there are 2 ginger cats. tabby:ginger = 3:2

c) Mrs Miggins has 12 tabby cats. How many ginger cats does she have?

Scale up the ratio from part b) to find the number of ginger cats.

tabby:ginger
$3:2 \xrightarrow{\times 4} = 12:8$
There are 8 ginger cats

Proportional Division

Grade 4

In a proportional division question a TOTAL AMOUNT is split into parts in a certain ratio. The key word here is PARTS — concentrate on 'parts' and it all becomes quite painless:

EXAMPLE: Jess, Mo and Greg share £9100 in the ratio 2:4:7. How much does Mo get?

1) **ADD UP THE PARTS:**
The ratio 2:4:7 means there will be a total of 13 parts: 2 + 4 + 7 = 13 parts

2) **DIVIDE TO FIND ONE "PART":**
Just divide the total amount by the number of parts: £9100 ÷ 13 = £700 (= 1 part)

3) **MULTIPLY TO FIND THE AMOUNTS:**
We want to know Mo's share, which is 4 parts: 4 parts = 4 × £700 = £2800

How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.
I will also use these at home to complete my own independent learning and revision of these key topics.

GLUE HERE

Year 10 Mathematics (Foundation): Low Stake Test scores (Autumn)

Tudor Values
Character Habits
Learning Habits

Topics	Date	Score
Pythagoras' Theorem, Forming and Solving Equations, Sharing Using ratio, Reverse percentages and Indices.		
Trigonometry, Simultaneous equations, Sharing using ratio, Estimation and Plotting linear graphs.		
Area and Circumference of a circle, Sharing using a ratio, Compound interest and depreciation, HCF & LCM using Prime factors and Expanding brackets.		
Pythagoras' Theorem, Forming and Solving Equations, Sharing Using ratio, Reverse percentages and Indices.		
Trigonometry, Simultaneous equations, Sharing using ratio, Estimation and Plotting linear graphs.		
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