Knowledge Organiser: Year 11 Foundation (1 – 3)

Terms (Grade 2)

Before you can do anything else with algebra, you must understand what a term is:

A TERM IS A COLLECTION OF NUMBERS, LETTERS AND BRACKETS, ALL MULTIPLIED/DIVIDED TOGETHER

Terms are separated by + and – signs. Every term has a + or – attached to the front of it.

If there's no sign in front of the first term, it means there's an invisible + sign.

$4x^2$ $+ 5x$ $- 2y$ $+ 6y^2$ $+ 4$

'x^2' term 'x' term 'y' term 'y^2' term 'number' term

1

Simplifying or *'Collecting Like Terms'* (Grade 2)

To simplify an algebraic expression made up of all the same terms, just add or subtract them.

EXAMPLES:

1. Simplify q + q + q + q + q

   Just add up all the q's:

   q + q + q + q + q = 5q

   'q' just means '1q'.

2. Simplify 4t + 5t – 2t

   Again, just combine the terms — don't forget there's a '–' before the 2t:

   4t + 5t – 2t = 7t

If you have a mixture of different terms, it's a bit more tricky. To simplify an algebraic expression like this, you combine 'like terms' (e.g. all the x terms, all the y terms, all the number terms etc.).

EXAMPLE:

Simplify $2x - 4 + 5x + 6$

Invisible + sign — number terms — x-terms

$(2x)\ (-4)\ (+5x)\ (+6) = (2x)\ (+5x)\ (-4)\ (+6) = 7x + 2$

$7x$ $+2$

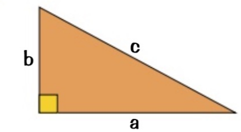
1) Put bubbles round each term — be sure you capture the +/– sign in front of each.
2) Then you can move the bubbles into the best order so that like terms are together.
3) Combine like terms.

Pythagoras' Theorem

Pythagoras' theorem sounds hard but it's actually dead simple. It's also dead important, so make sure you really get your teeth into it.

Pythagoras' Theorem — $a^2 + b^2 = c^2$ (Grade 4)

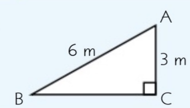
1) PYTHAGORAS' THEOREM only works for RIGHT-ANGLED TRIANGLES.
2) Pythagoras uses two sides to find the third side.
3) The BASIC FORMULA for Pythagoras is $a^2 + b^2 = c^2$
4) Make sure you get the numbers in the RIGHT PLACE. c is the longest side (called the hypotenuse) and it's always opposite the right angle.
5) Always CHECK that your answer is SENSIBLE.



$a^2 + b^2 = c^2$

EXAMPLE:

ABC is a right-angled triangle.
AB = 6 m and AC = 3 m.
Find the exact length of BC.



It's not always c you need to find — loads of people go wrong here.

1) Write down the formula. $a^2 + b^2 = c^2$
2) Put in the numbers. $BC^2 + 3^2 = 6^2$
3) Rearrange the equation. $BC^2 = 6^2 - 3^2 = 36 - 9 = 27$
4) Take square roots to find BC. $BC = \sqrt{27} = 3\sqrt{3}$ m
5) 'Exact length' means you should give your answer as a surd — simplified if possible.

Remember to check the answer's sensible — here it's about 5.2, which is between 3 and 6, so that seems about right...

2

Two-Way Tables Show *How Many* in each *Category* (Grade 2)

EXAMPLE: This table shows the number of cakes and pies a bakery sold on Friday and Saturday.

	Cakes	Pies	Total
Friday	12	10	
Saturday	4	14	18
Total	16	24	40

a) How many pies were sold on Saturday?

Read across to 'Pies' and down to 'Saturday'. **14 pies**

b) How many items were sold in total on Friday?

Add the number of cakes for Friday to the number of pies. **12 + 10 = 22**

Or you could subtract the total for Saturday from the overall total: 40 – 18 = 22.

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+	+	makes	+
+	–	makes	–
–	+	makes	–
–	–	makes	+

Vectors — Theory

4

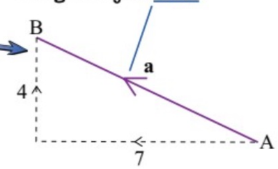
Vectors represent a movement of a certain size in a certain direction.
They might seem a bit weird at first, but there are really just a few facts to get to grips with...

The Vector Notations (Grade 5)

There are several ways to write vectors...

1) Column vectors: $\begin{pmatrix}2\\-5\end{pmatrix}$ — 2 units right, — 5 units down; $\begin{pmatrix}-7\\4\end{pmatrix}$ — 7 units left, — 4 units up
2) **a** —— exam questions use bold like this
3) $\underset{\sim}{a}$ —— you should always underline them
4) $\overrightarrow{AB}$ —— this means the vector from point A to point B

They're shown on a diagram by an arrow.

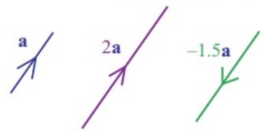


Multiplying a Vector by a Number (Grade 5)

1) Multiplying a vector by a positive number changes the vector's size but not its direction.

2) If the number's negative then the direction gets switched.

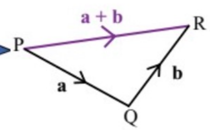
Vectors that are multiples of each other are parallel.



Adding and Subtracting Vectors (Grade 5)

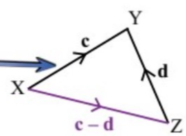
You can describe movements between points by adding and subtracting known vectors.

"$\underset{\sim}{a}$ + $\underset{\sim}{b}$" means 'go along $\underset{\sim}{a}$ then $\underset{\sim}{b}$'.



In the diagrams, $\overrightarrow{PR} = \underset{\sim}{a} + \underset{\sim}{b}$ and $\overrightarrow{XZ} = \underset{\sim}{c} - \underset{\sim}{d}$.

"$\underset{\sim}{c}$ – $\underset{\sim}{d}$" means 'go along c then backwards along d' (the minus sign means go the opposite way).



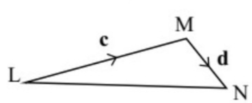
When adding column vectors, add the top to the top and the bottom to the bottom. The same goes when subtracting. $\begin{pmatrix}3\\-1\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}3+5\\-1+3\end{pmatrix}=\begin{pmatrix}8\\2\end{pmatrix}$

Vector directions — In 5p + 2q take a left onto AB...

There are a lot of different ideas to take in on this page, so here's a bit of a warm-up to get you into the swing of things before the proper examples on the next page.

Q1 Calculate $\begin{pmatrix}4\\7\end{pmatrix}-\begin{pmatrix}3\\2\end{pmatrix}$. [1 mark] (Grade 5)

Q2 Find $\overrightarrow{LN}$ on the diagram to the right. [1 mark] (Grade 5)



Record Results in Frequency Trees (Grade 4)

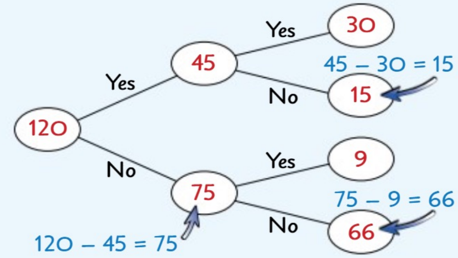
5

When an experiment has two or more steps, you can record the results using a frequency tree.

EXAMPLE: 120 GCSE maths students were asked if they would go on to do A-level maths.

- 45 of them said they would go on to do A-level maths.
- 30 of the students who said they would do A-level maths actually did.
- 9 of the students who said they wouldn't do A-level maths actually did.

a) Complete the frequency tree below.



b) A student who said they wouldn't do A-level maths is chosen at random. What is the probability that they did do A-level maths?

9 out of the 75 students who said they wouldn't do A-level maths actually did.

So the probability is $\frac{9}{75}$ = 0.12

Multiplying Brackets (Grade 3)

6

The key thing to remember about multiplying brackets is that the thing outside the brackets multiplies each separate term inside the brackets.

EXAMPLE: Expand the following:

a) $3(2x + 5)$
$= (3 \times 2x) + (3 \times 5)$
$= 6x + 15$

b) $-4(3y - 2)$
$= (-4 \times 3y) + (-4 \times -2)$
$= -12y + 8$

c) $2e(e - 4)$
$= (2e \times e) + (2e \times -4)$
$= 2e^2 - 8e$

EXAMPLE: Expand $x(2x + 1) + y(y - 4) + 3x(y + 2)$

1) Expand each bracket separately. $x(2x + 1) + y(y - 4) + 3x(y + 2)$
$= 2x^2 + x + y^2 - 4y + 3xy + 6x$

2) Group together like terms. $= 2x^2 + x + 6x + 3xy + y^2 - 4y$

3) Simplify the expression. $= 2x^2 + 7x + 3xy + y^2 - 4y$

3) Multiplying

Multiply top and bottom separately. Then simplify your fraction as far as possible.

EXAMPLE: Find $\frac{8}{5} \times \frac{7}{12}$.

Multiply the top and bottom separately: $\frac{8}{5} \times \frac{7}{12} = \frac{8 \times 7}{5 \times 12}$

Then simplify — top and bottom both divide by 4. $= \frac{56}{60} = \frac{14}{15}$

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4) Dividing

Turn the 2nd fraction UPSIDE DOWN and then multiply:

When you're multiplying or dividing with mixed numbers, always turn them into improper fractions first.

EXAMPLE: Find $2\frac{1}{3} \div 3\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{3} \div 3\frac{1}{2} = \frac{7}{3} \div \frac{7}{2}$

Turn $\frac{7}{2}$ upside down and multiply: $= \frac{7}{3} \times \frac{2}{7} = \frac{7 \times 2}{3 \times 7}$

Simplify — top and bottom both divide by 7. $= \frac{14}{21} = \frac{2}{3}$

6) Adding, subtracting — sort the denominators first

1) Make sure the denominators are the same (see above).
2) Add (or subtract) the top lines only.

If you're adding or subtracting mixed numbers, it usually helps to convert them to improper fractions first.

EXAMPLE: Calculate $2\frac{1}{5} - 1\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{5} - 1\frac{1}{2} = \frac{11}{5} - \frac{3}{2}$

Find a common denominator: $= \frac{22}{10} - \frac{15}{10}$

Combine the top lines: $= \frac{22 - 15}{10} = \frac{7}{10}$

Powers

Powers are a very Useful Shorthand

1. Powers are 'numbers multiplied by themselves so many times': $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$ ('two to the power 7')
2. The powers of ten are really easy — the power tells you the number of zeros: $10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^6 = 1\,000\,000$ (to the power of 6; 6 zeros)
3. Use the $x^{\blacksquare}$ or y^x button on your calculator to find powers, e.g. press 3 · 7 $x^{\blacksquare}$ 3 = to get $3.7^3 = 50.653$.
4. Anything to the power 1 is just itself, e.g. $4^1 = 4$.
5. 1 to any power is still 1, e.g. $1^{457} = 1$.
6. Anything to the power 0 is just 1, e.g. $5^0 = 1$, $67^0 = 1$, $x^0 = 1$.

Four Easy Rules:

1) When MULTIPLYING, you ADD THE POWERS. e.g. $3^4 \times 3^6 = 3^{4+6} = 3^{10}$
2) When DIVIDING, you SUBTRACT THE POWERS. e.g. $c^4 \div c^2 = c^{4-2} = c^2$
3) When RAISING one power to another, you MULTIPLY THE POWERS. e.g. $(3^2)^4 = 3^{2 \times 4} = 3^8$
4) FRACTIONS — Apply the power to both TOP and BOTTOM. e.g. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

EXAMPLE: $a = 5^9$ and $b = 5^4 \times 5^2$. What is the value of $\frac{a}{b}$?

1) Work out b — add the powers: $b = 5^4 \times 5^2 = 5^{4+2} = 5^6$

2) Divide a by b — subtract the powers: $\frac{a}{b} = 5^9 \div 5^6 = 5^{9-6} = 5^3 = 125$

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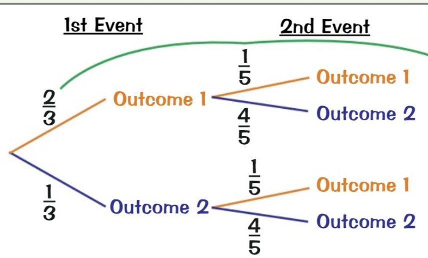
Tree Diagrams

On any set of branches which meet at a point, the probabilities must add up to 1.



$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

$\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

Total = 1

2) Multiply along the branches to get the end probabilities.

3) Check your diagram — the end probabilities must add up to 1.

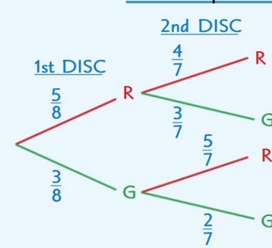
4) To answer any question, add up the relevant end probabilities (see below).

9

A good way to deal with conditional probability questions is to draw a tree diagram. The probabilities on a set of branches will change depending on the previous event.

This example was done 'with replacement' on p.111.

EXAMPLE: A box contains 5 red discs and 3 green discs. Two discs are taken at random without replacement. Find the probability that both discs are the same colour.



The probabilities for the 2nd pick depend on the colour of the 1st disc picked. This is because the 1st disc is not replaced.

P(both discs are red) = P(R and R) = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

P(both discs are green) = P(G and G) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

P(both discs are same colour) = P(R and R or G and G) $= \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$

How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.
I will also use these at home to complete my own independent learning and revision of these key topics.

GLUE HERE

Year 11 - Mathematics (Foundation 1-2): Low Stake Test scores (Autumn)

Tudor Values
Character Habits
Learning Habits

Topics	Date	Score
Collecting Like Terms, Laws of Indices, Pythagoras' Theorem (Hypotenuse), Adding and Subtracting Column Vectors and Two-Way Tables.		
Expanding Brackets, Pythagoras' Theorem (Shorter), Multiplying a Vector by a Scalar, Frequency Trees, Tree Diagrams and Four Operations with Fractions.		
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