Knowledge Organiser: Year 11 (F)

1

Angles in Polygons

A polygon is a many-sided shape, and can be regular or irregular. A regular polygon (p.72) is one where all the sides and angles are the same. By the end of this page you'll be able to work out the angles in them. Wowzers.

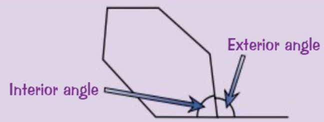
Exterior and Interior Angles (Grade 3)

You need to know what exterior and interior angles are and how to find them.

For ANY POLYGON (regular or irregular):

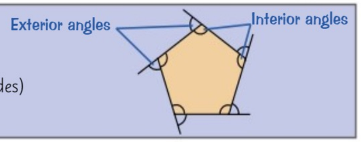
SUM OF EXTERIOR ANGLES = 360°

INTERIOR ANGLE = 180° – EXTERIOR ANGLE



For REGULAR POLYGONS only:

EXTERIOR ANGLE $= \dfrac{360°}{n}$ (n is the number of sides)



EXAMPLE: Find the exterior and interior angles of a regular octagon.

Octagons have 8 sides: exterior angle $= \dfrac{360°}{n} = \dfrac{360°}{8} = 45°$

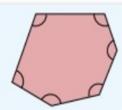
Use the exterior angle to find the interior angle: interior angle = 180° – exterior angle
= 180° – 45° = 135°

The Tricky One — Sum of Interior Angles (Grade 3)

This formula for the sum of the interior angles works for ALL polygons, even irregular ones:

SUM OF INTERIOR ANGLES = (n – 2) × 180°

EXAMPLE: Find the sum of the interior angles of the polygon on the right.

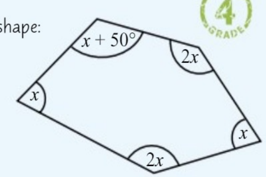


The polygon is a hexagon, so n = 6: Sum of interior angles = (n – 2) × 180°
= (6 – 2) × 180° = 720°

Don't panic if those pesky examiners put algebra in an interior angle question. It looks worse than it is.

EXAMPLE: (Grade 4) Find the value of x in the diagram on the right.

First, find the sum of the interior angles of the 5-sided shape:



Sum of interior angles = (n – 2) × 180°
= (5 – 2) × 180° = 540°

Now write an equation and solve it to find x:

$2x + x + 2x + x + (x + 50°) = 540°$

$7x + 50° = 540° \rightarrow 7x = 490° \rightarrow x = 70°$

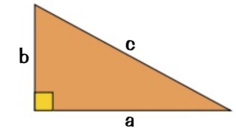
2

Pythagoras' Theorem

Pythagoras' theorem sounds hard but it's actually dead simple.
It's also dead important, so make sure you really get your teeth into it.

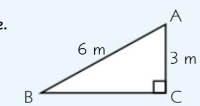
Pythagoras' Theorem — $a^2 + b^2 = c^2$ (Grade 4)

1) PYTHAGORAS' THEOREM only works for RIGHT-ANGLED TRIANGLES.
2) Pythagoras uses two sides to find the third side.
3) The BASIC FORMULA for Pythagoras is $a^2 + b^2 = c^2$
4) Make sure you get the numbers in the RIGHT PLACE. c is the longest side (called the hypotenuse) and it's always opposite the right angle.
5) Always CHECK that your answer is SENSIBLE.



$a^2 + b^2 = c^2$

EXAMPLE: ABC is a right-angled triangle.
AB = 6 m and AC = 3 m.
Find the exact length of BC.



It's not always c you need to find — loads of people go wrong here.

1) Write down the formula. $a^2 + b^2 = c^2$
2) Put in the numbers. $BC^2 + 3^2 = 6^2$
3) Rearrange the equation. $BC^2 = 6^2 - 3^2 = 36 - 9 = 27$
4) Take square roots to find BC. $BC = \sqrt{27} = 3\sqrt{3}$ m
5) 'Exact length' means you should give your answer as a surd — simplified if possible.

Remember to check the answer's sensible — here it's about 5.2, which is between 3 and 6, so that seems about right...

3

Best Buy Questions (Grade 3)

A slightly different type of direct proportion question is comparing the 'value for money' of 2 or 3 similar items. For these, follow the second GOLDEN RULE...

Divide by the PRICE in pence (to get the amount per penny)

EXAMPLE: The local 'Supplies 'n' Vittals' stocks two sizes of Jamaican Gooseberry Jam, as shown on the right. Which of these represents better value for money?

350 g at 80p 100 g at 42p

Follow the GOLDEN RULE — divide by the price in pence to get the amount per penny.

In the 350 g jar you get 350 g ÷ 80p = 4.38 g per penny
In the 100 g jar you get 100 g ÷ 42p = 2.38 g per penny

The 350 g jar is better value for money, because you get more jam per penny.

Formulas and Equations from Words

Sometimes, you might be asked to use an expression to solve an equation.

EXAMPLE: A zoo has x zebras and four times as many lemurs. The difference between the number of zebras and the number of lemurs is 45. How many zebras does the zoo have?

The zoo has x zebras and $4 \times x = 4x$ lemurs.
The difference is $4x - x = 3x$, so $3x = 45$, which means $x = 15$.
So the zoo has 15 zebras.

Once you've formed the equation, you need to solve it to find the value of x.

EXAMPLE: Will, Naveed and Camille give some books to charity. Naveed gives 6 more books than Will, and Camille gives 7 more books than Naveed. Between them, they give away 46 books. How many books did they give each?

Let the number of books Will gives be x.
Then Naveed gives $x + 6$ books
and Camille gives $(x + 6) + 7 = x + 13$ books
So in total they give $x + x + 6 + x + 13 = 3x + 19$ books

So $3x + 19 = 46$
$3x = 27$
$x = 9$

You're told this in the question.

So Will gives 9 books,
Naveed gives $9 + 6 = 15$ books and
Camille gives $15 + 7 = 22$ books.

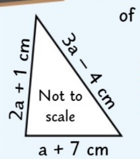
Use Shape Properties to Find Formulas and Equations

GRADE 4

In some questions, you'll need to use what you know about shapes (e.g. side lengths or areas) to come up with a formula or an equation to solve.

EXAMPLE:

a) Write a formula for P, the perimeter of the triangle below, in terms of a.



Form an expression for the perimeter:
$P = (a + 7) + (2a + 1) + (3a - 4)$
$P = 6a + 4$ cm

b) If the triangle has a perimeter of 58 cm, find the value of a.

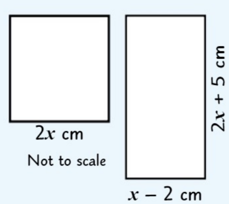
P = 58, so set your formula equal to 58 and solve to find a:
$6a + 4 = 58$
$6a = 54$
$a = 9$

Compare Dimensions of Two Shapes to Find Equations

GRADE 5

You might get a question that involves two shapes with related areas or perimeters — you'll have to use this fact to find side lengths or missing values.

EXAMPLE: The perimeter of the rectangle is the same as the perimeter of the square. Find the value of x.



Perimeter of square $= 2x + 2x + 2x + 2x = 8x$ cm
Perimeter of rectangle $= (2x + 5) + (x - 2) + (2x + 5) + (x - 2)$
$= 6x + 6$ cm

Set the perimeter of the rectangle equal to the perimeter of the square and solve:
$8x = 6x + 6$
$2x = 6$
$x = 3$

4

Trigonometry — Examples

Here are some lovely examples using the method from p.96 to help you through the trials of trig.

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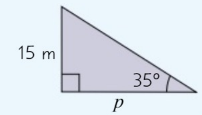
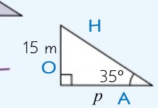
Examples:

GRADE 6

1 Find the length of p in the triangle shown to 3 s.f.





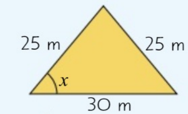
1) Label the sides
2) Write down — SOH CAH TOA
3) O and A involved
4) Write down the formula triangle

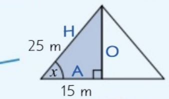
5) You want A so cover it up to give — $A = \frac{O}{T}$
6) Put in the numbers — $p = \frac{15}{\tan 35°} = 21.422...$
$= 21.4$ m (3 s.f.)

Is it sensible? Yes, it's a bit bigger than 15, as the diagram suggests.

2 Find the angle x in this triangle to 1 d.p.

It's an isosceles triangle so split it down the middle to get a right-angled triangle.





1) Label the sides
2) Write down — SOH CAH TOA
3) A and H involved
4) Write down the formula triangle

5) You want the angle so cover up C to give — $C = \frac{A}{H}$
6) Put in the numbers — $\cos x = \frac{15}{25} = 0.6$
7) Find the inverse — $\Rightarrow x = \cos^{-1}(0.6) = 53.1301...°$
$= 53.1°$ (1 d.p.)

Is it sensible? Yes, the angle looks about 50°.

Percentages

Type 6 — Finding the Original Value

GRADE 4

This is the type that most people get wrong — but only because they don't recognise it as this type, and don't apply this simple method:

1) Write the amount in the question as a percentage of the original value.
2) Divide to find 1% of the original value.
3) Multiply by 100 to give the original value (= 100%).

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EXAMPLE: A house increases in value by 10% to £165 000. Find what it was worth before the rise.

Note: The new, not the original value is given.

1) An increase of 10% means £165 000 represents 110% of the original value.
2) Divide by 110 to find 1% of the original value.
3) Then multiply by 100.

$$\begin{aligned} \div 110 \quad & £165\,000 = 110\% \\ & £1500 = 1\% \\ \times 100 \quad & £150\,000 = 100\% \end{aligned}$$

If it was a decrease of 10%, then you'd put '£165 000 = 90%' and divide by 90 instead of 110.

So the original value was £150 000

Always set them out exactly like this example. The trickiest bit is deciding the top % figure on the right-hand side — the 2nd and 3rd rows are always 1% and 100%.

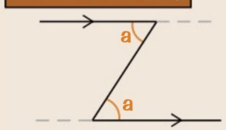
Alternate, Allied and *Corresponding Angles*

GRADE 3

Watch out for these 'Z', 'C', 'U' and 'F' shapes popping up.
They're a dead giveaway that you've got a pair of parallel lines.

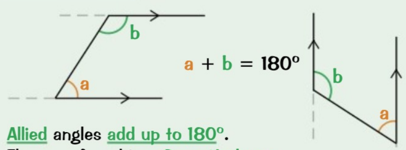
Don't call them Z, C, U and F angles in the exam — you'll need to use their proper names.

ALTERNATE ANGLES



Alternate angles are the same.
They are found in a Z-shape.

ALLIED ANGLES



Allied angles add up to 180°.
They are found in a C- or U-shape.

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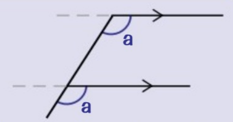
CORRESPONDING ANGLES

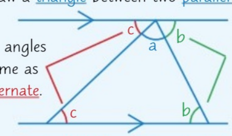


Corresponding angles are the same.
They are found in an F-shape.

EXAMPLE: Prove that the angles in a triangle add up to 180°.

This is the proof of rule 1 from the previous page.
First, draw a triangle between two parallel lines:

These two angles are the same as they're alternate.



These two angles are the same as they're alternate.

Angles on a straight line add up to 180°, so a + b + c = 180°.

Estimating Calculations

GRADE 4

Have a look at the previous page to remind yourself how to round to 1 s.f.

1. Round everything off to 1 significant figure.
2. Then work out the answer using these nice easy numbers.
3. Show all your working or you won't get the marks.

EXAMPLES: 1. Estimate the value of 42.6 × 12.1.

≈ means 'approximately equal to'.

1) Round each number to 1 s.f. $42.6 \times 12.1 \approx 40 \times 10$
2) Do the calculation with the rounded numbers. $= 400$

You might have to say if it's an underestimate or an overestimate. Here, you rounded both numbers down, so it's an underestimate.

2. Estimate the value of $\frac{\sqrt{6242 \div 57}}{9.8 - 4.7}$.

Don't be put off by the square root, just round each number to 1 s.f. and do the calculation.

$$\frac{\sqrt{6242 \div 57}}{9.8 - 4.7} \approx \frac{\sqrt{6000 \div 60}}{10 - 5} = \frac{\sqrt{100}}{5} = \frac{10}{5} = 2$$

3. Jo has a cake-making business. She spent £984.69 on flour last year. A bag of flour costs £1.89, and she makes an average of 5 cakes from each bag of flour. Work out an estimate of how many cakes she made last year.

1) Estimate number of bags of flour — round numbers to 1 s.f.

$$\text{Number of bags of flour} = \frac{984.69}{1.89} \approx \frac{1000}{2} = 500$$

2) Multiply to find the number of cakes.

$$\text{Number of cakes} \approx 500 \times 5 = 2500$$

Don't panic if you get a 'real-life' estimating question — just round everything to 1 s.f. as before.

Powers

You've already seen 'to the power 2' and 'to the power 3' — they're just 'squared' and 'cubed'.
They're just the tip of the iceberg — any number can be a power if it puts its mind to it...

Powers are a very Useful Shorthand

GRADE 4

1. Powers are 'numbers multiplied by themselves so many times': $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$ ('two to the power 7')
2. The powers of ten are really easy — the power tells you the number of zeros: to the power of 6

   $10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^6 = 1000000$ 6 zeros
3. Use the $x^{\blacksquare}$ or y^x button on your calculator to find powers, e.g. press 3 · 7 $x^{\blacksquare}$ 3 = to get $3.7^3 = 50.653$.
4. Anything to the power 1 is just itself, e.g. $4^1 = 4$.
5. 1 to any power is still 1, e.g. $1^{457} = 1$.
6. Anything to the power 0 is just 1, e.g. $5^0 = 1$, $67^0 = 1$, $x^0 = 1$.

Four Easy Rules:

GRADE 4

1) When MULTIPLYING, you ADD THE POWERS. e.g. $3^4 \times 3^6 = 3^{4+6} = 3^{10}$

2) When DIVIDING, you SUBTRACT THE POWERS. e.g. $c^4 \div c^2 = c^{4-2} = c^2$

3) When RAISING one power to another, you MULTIPLY THE POWERS. e.g. $(3^2)^4 = 3^{2\times4} = 3^8$

4) FRACTIONS — Apply the power to both TOP and BOTTOM. e.g. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

EXAMPLE: $a = 5^9$ and $b = 5^4 \times 5^2$. What is the value of $\frac{a}{b}$?

1) Work out b — add the powers: $b = 5^4 \times 5^2 = 5^{4+2} = 5^6$

2) Divide a by b — subtract the powers: $\frac{a}{b} = 5^9 \div 5^6 = 5^{9-6} = 5^3 = 125$

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One Trickier Rule

GRADE 5

To find a negative power — turn it upside-down.

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

E.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

Fractions without a Calculator

3) Multiplying

Multiply top and bottom separately. Then simplify your fraction as far as possible.

EXAMPLE: Find $\frac{8}{5} \times \frac{7}{12}$.

Multiply the top and bottom separately: $\frac{8}{5} \times \frac{7}{12} = \frac{8 \times 7}{5 \times 12}$

Then simplify — top and bottom both divide by 4. $= \frac{56}{60} = \frac{14}{15}$

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4) Dividing

Turn the 2nd fraction UPSIDE DOWN and then multiply:

When you're multiplying or dividing with mixed numbers, always turn them into improper fractions first.

EXAMPLE: Find $2\frac{1}{3} \div 3\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{3} \div 3\frac{1}{2} = \frac{7}{3} \div \frac{7}{2}$

Turn $\frac{7}{2}$ upside down and multiply: $= \frac{7}{3} \times \frac{2}{7} = \frac{7 \times 2}{3 \times 7}$

Simplify — top and bottom both divide by 7. $= \frac{14}{21} = \frac{2}{3}$

6) Adding, subtracting — sort the denominators first

1) Make sure the denominators are the same (see above).
2) Add (or subtract) the top lines only.

If you're adding or subtracting mixed numbers, it usually helps to convert them to improper fractions first.

EXAMPLE: Calculate $2\frac{1}{5} - 1\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{5} - 1\frac{1}{2} = \frac{11}{5} - \frac{3}{2}$

Find a common denominator: $= \frac{22}{10} - \frac{15}{10}$

Combine the top lines: $= \frac{22 - 15}{10} = \frac{7}{10}$

Show Sets on Venn Diagrams

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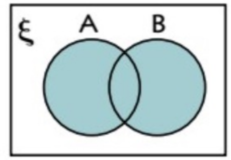
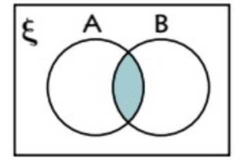
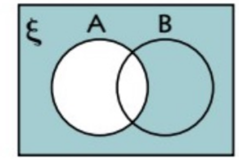
1) On a Venn diagram, each set is represented by a circle. The universal set is everything inside the rectangle.
2) The diagram can show either the actual elements of each set, or the number of elements in each set.



The union of sets A and B (written A ∪ B) contains all the elements in either set A or set B — it's everything inside the circles.



The intersection of sets A and B (written A ∩ B) contains all the elements in both set A and set B — it's where the circles overlap.

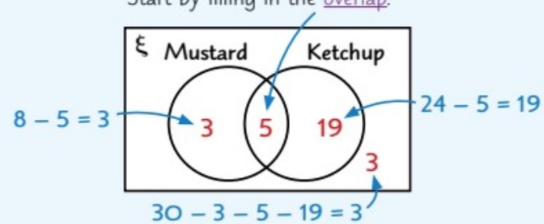


The complement of set A (written A') contains all members of the universal set that aren't in set A — it's everything outside circle A.

EXAMPLE: In a class of 30 pupils, 8 of them like mustard, 24 of them like ketchup and 5 of them like both mustard and ketchup.

a) Complete the Venn diagram below showing this information.

Start by filling in the overlap.



b) How many pupils like mustard or ketchup?

This is the number of pupils in the union of the two sets. $3 + 5 + 19 = 27$

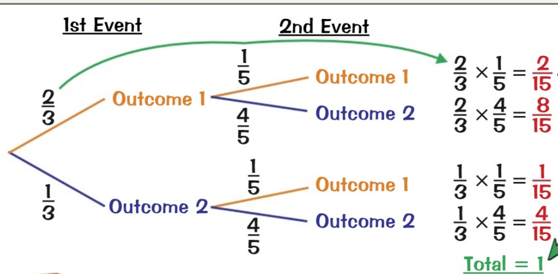
c) What is the probability that a randomly selected pupil will like mustard and ketchup?

5 out of 30 pupils are in the intersection. $\frac{5}{30} = \frac{1}{6}$

This is P(M ∩ K).

Tree Diagrams

On any set of branches which meet at a point, the probabilities must add up to 1.



2) Multiply along the branches to get the end probabilities.

3) Check your diagram — the end probabilities must add up to 1.

4) To answer any question, add up the relevant end probabilities (see below).

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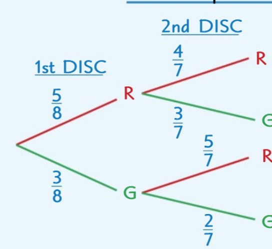
A good way to deal with conditional probability questions is to draw a tree diagram. The probabilities on a set of branches will change depending on the previous event.

This example was done 'with replacement' on p.111.

EXAMPLE: A box contains 5 red discs and 3 green discs. Two discs are taken at random without replacement. Find the probability that both discs are the same colour.



The probabilities for the 2nd pick depend on the colour of the 1st disc picked. This is because the 1st disc is not replaced.

P(both discs are red) = P(R and R) = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

P(both discs are green) = P(G and G) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

P(both discs are same colour) = P(R and R or G and G) $= \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$

How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

Year 11 Mathematics (Foundation): Low Stake Test scores (Autumn)

Tudor Values
Character Habits
Learning Habits

Topics	Date	Score
Pythagoras’ theorem, sharing using ratio, estimation, simplifying expressions using the laws of indices and four operations with fractions.		
Trigonometry, best buys, tree diagrams, angles on parallel lines and reverse percentages.		
Forming and solving equations, exchange rates, highest common factor, lowest common multiple, Venn diagrams and angles in polygons.		
Pythagoras’ theorem, sharing using ratio, estimation, simplifying expressions using the laws of indices and four operations with fractions.		
Trigonometry, best buys, tree diagrams, angles on parallel lines and reverse percentages.		
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