Knowledge Organiser
Year 11 Foundation (Spring)

1

Learn the Golden Rule for Proportion Questions (Grade 3)

There are lots of exam questions which at first sight seem completely different but in fact they can all be done using the GOLDEN RULE...

DIVIDE FOR ONE, THEN TIMES FOR ALL

EXAMPLE: 5 pints of milk cost £1.30. How much will 3 pints cost?

The GOLDEN RULE tells you to:
Divide the price by 5 to find how much FOR ONE PINT, then multiply by 3 to find how much FOR 3 PINTS.

1 pint: £1.30 ÷ 5 = 0.26 = 26p
3 pints: 26p × 3 = 78p

EXAMPLE: Emma is handing out some leaflets. She gets paid per leaflet she hands out. If she hands out 300 leaflets she gets £2.40. How many leaflets will she have to hand out to earn £8.50?

Divide by £2.40 to find how many leaflets she has to hand out to earn £1.

To earn £1: 300 ÷ £2.40 = 125 leaflets

Multiply by £8.50 to find how many leaflets she has to hand out to earn £8.50.

To earn £8.50: 125 × £8.50 = 1062.5
So she'll need to hand out 1063 leaflets.

You need to round your answer up because 1062 wouldn't be enough.

Scaling Recipes Up or Down (Grade 3)

EXAMPLE: Judy is making orange and pineapple punch using the recipe shown on the right. She wants to make enough to serve 20 people. How much of each ingredient will Judy need?

Fruit Punch (serves 8)
800 ml orange juice
140 g fresh pineapple

The GOLDEN RULE tells you to divide each amount by 8 to find how much FOR ONE PERSON, then multiply by 20 to find how much FOR 20 PEOPLE.

So for 1 person you need:
800 ml ÷ 8 = 100 ml orange juice ⇒
140 g ÷ 8 = 17.5 g pineapple ⇒

And for 20 people you need:
20 × 100 ml = 2000 ml orange juice
20 × 17.5 g = 350 g pineapple

2

Six Steps for Simultaneous Equations (Grade 5)

EXAMPLE: Solve the simultaneous equations $2x + 4y = 6$
$4x + 3y = -3$

Both your equations should be in the form $ax + by = c$, where a, b and c are numbers.

1. Label your equations ① and ②.

$2x + 4y = 6$ — ①
$4x + 3y = -3$ — ②

2. Match up the numbers in front of either the x's or y's in both equations. You may need to multiply one or both equations by a suitable number. Relabel the equations ③ and ④ if you need to change them.

① × 2: $4x + 8y = 12$ — ③
$4x + 3y = -3$ — ②

You don't need to change equation 2 for this example.

3. Add or subtract the two equations to eliminate the terms with the same number in front.

③ − ②: $4x + 8y = 12$
$-\ 4x + 3y = -3$
$0x + 5y = 15$

If the numbers have the same sign (both +ve or both −ve) then subtract. If the numbers have opposite signs (one +ve and one −ve) then add.

4. Solve the resulting equation.

$5y = 15 \Rightarrow y = 3$

5. Substitute the value you've found back into equation ① and solve it.

Sub $y = 3$ into ①: $2x + (4 \times 3) = 6 \Rightarrow 2x + 12 = 6 \Rightarrow 2x = -6 \Rightarrow x = -3$

6. Substitute both these values into equation ② to make sure it works. If it doesn't then you've done something wrong and you'll have to do it all again.

Sub x and y into ②: $(4 \times -3) + (3 \times 3) = -12 + 9 = -3$, which is right, so it's worked.
So the solutions are: $x = -3$, $y = 3$

3

Find Averages from Grouped Frequency Tables (Grade 5)

Add extra columns for 'mid-interval value' and 'frequency × mid-interval value'. Add up the values in the 4th column to estimate the total weight of the 60 children.

Weight (w kg)	Frequency	Mid-interval value	Frequency × mid-interval value
$30 < w \le 40$	8	35	280
$40 < w \le 50$	16	45	720
$50 < w \le 60$	18	55	990
$60 < w \le 70$	12	65	780
$70 < w \le 80$	6	75	450
Total	60	—	3220

$$\text{Mean} \approx \frac{\text{total weight}}{\text{number of children}}$$

total weight ← 4th column total
number of children ← 2nd column total

$$= \frac{3220}{60} = 53.7 \text{ kg (3 s.f.)}$$

Compound Growth and Decay

One more sneaky % type for you... Unlike simple interest, in compound interest the amount added on (or taken away) changes each time — it's a percentage of the new amount, rather than the original amount.

The Formula (Grade 5)

This topic is simple if you LEARN THIS FORMULA. If you don't, it's pretty well impossible:

$$N = N_0 \times (\text{multiplier})^n$$

Amount after n days/hours/years

Number of days/hours/years

Initial amount

Percentage change multiplier
E.g. 5% increase is 1.05 (= 1 + 0.05)
26% decrease is 0.74 (= 1 – 0.26)

3 Examples to show you how EASY it is: (Grade 5)

Compound interest is a popular context for these questions — it means the interest is added on each time, and the next lot of interest is calculated using the new total rather than the original amount.

EXAMPLE: Daniel invests £1000 in a savings account which pays 8% compound interest per annum. How much will there be after 6 years?

Use the formula: Amount = $1000(1.08)^6$ = £1586.87

initial amount — 8% increase — 6 years

'Per annum' just means 'each year'.

Depreciation questions are about things (e.g. cars) which decrease in value over time.

EXAMPLE: Susan has just bought a car for £6500.

a) If the car depreciates by 9% each year, how much will it be worth in 3 years' time?

Use the formula: Value = $6500(0.91)^3$ = £4898.21

b) How many complete years will it be before the car is worth less than £3000?

Use the formula again but this time you know don't know n.

Value = $6500(0.91)^n$

Use trial and error to find how many years it will be before the value drops below £3000.

If n = 8, $6500(0.91)^8$ = 3056.6414....
n = 9, $6500(0.91)^9$ = 2781.5437...

It will be 9 years before the car is worth less than £3000.

4

Doing the 'Table of Values' (Grade 3)

5

EXAMPLE: Draw the graph of $y = 2x - 3$ for values of x from –2 to 4.

1. Choose 3 easy x-values for your table:
Use x-values from the grid you're given.
Avoid negative ones if you can.

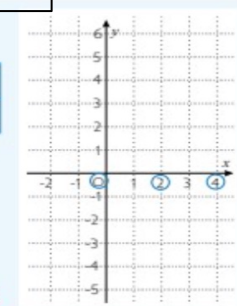
x	0	2	4
y			

2. Find the y-values by putting each x-value into the equation:

x	0	2	4
y	–3	1	5

When $x = 0$,
$y = 2x - 3$
$= (2 \times 0) - 3 = -3$

When $x = 4$,
$y = 2x - 3$
$= (2 \times 4) - 3 = 5$



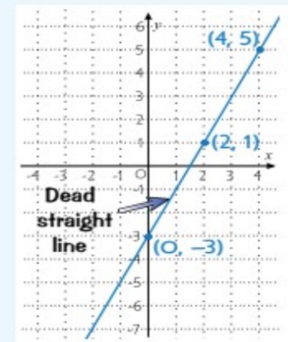
Plotting the Points and Drawing the Graph (Grade 3)

EXAMPLE: ...continued from above.

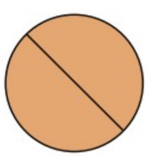
3. PLOT EACH PAIR of x- and y- values from your table.
The table gives the coordinates (0, –3), (2, 1) and (4, 5).

Now draw a STRAIGHT LINE through your points.

If one point looks a bit wacky, check 2 things:
– the y-values you worked out in the table
– that you've plotted the points properly.



Area and Circumference of Circles (Grade 3)



Area of circle = $\pi \times (\text{radius})^2$
Remember that the radius is half the diameter.

$$A = \pi r^2$$

For these formulas, use the π button on your calculator. For non-calculator questions, use $\pi \approx 3.142$.

Circumference = $\pi \times$ diameter
= $2 \times \pi \times$ radius

$$C = \pi D = 2\pi r$$

6

Finding the nth Term of a Sequence (Grade 4)

7

This method works for sequences with a common difference — where you add or subtract the same number each time.

EXAMPLE: Find an expression for the nth term of the sequence that starts 5, 8, 11, 14, ...

n:	1	2	3	4
term:	5	8	11	14
	+3	+3	+3	
3n:	3	6	9	12
	+2	+2	+2	+2
term:	5	8	11	14

So the expression for the nth term is 3n + 2

1. Find the common difference. It's 3, so this tells you '3n' is in the formula.
2. List the values of 3n.
3. Work out what you have to add or subtract to get from 3n to the term. So it's +2.
4. Put '3n' and '+2' together.

Check your formula by putting the first few values of n back in:
n = 1 gives 3n + 2 = 3 + 2 = 5 ✓
n = 2 gives 3n + 2 = 6 + 2 = 8 ✓

Use the FOIL Method to Multiply Out Double Brackets

There's a handy way to multiply double brackets — it's called the FOIL method and works like this: (Grade 4)

First — multiply the first term in each bracket together
Outside — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)
Inside — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)
Last — multiply the second term in each bracket together

When multiplying double brackets, you get 4 terms — and 2 of them usually combine to leave 3 terms.

EXAMPLES:

1. Expand and simplify $(x + 3)(x + 8)$

$$(x+3)(x+8) = \overset{F}{(x \times x)} + \overset{O}{(x \times 8)} + \overset{I}{(3 \times x)} + \overset{L}{(3 \times 8)}$$
$$= x^2 + 8x + 3x + 24$$
$$= x^2 + 11x + 24$$

The two x-terms combine together.

2. Expand and simplify $(n - 2)(2n + 7)$

$$(n-2)(2n+7) = \overset{F}{(n \times 2n)} + \overset{O}{(n \times 7)} + \overset{I}{(-2 \times 2n)} + \overset{L}{(-2 \times 7)}$$
$$= 2n^2 + 7n - 4n - 14$$
$$= 2n^2 + 3n - 14$$

Speed = Distance ÷ Time

(Grade 3)

Speed is the distance travelled per unit time — the number of km per hour or metres per second.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \qquad \text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}} \qquad \text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

A formula triangle is a mighty handy tool for remembering formulas. Here's the one for speed.
To remember the order of the letters (S^DT) we have the words SaD Times.
So if it's a question on speed, distance and time, just say SAD TIMES.

D
S × T

HOW DO YOU USE FORMULA TRIANGLES?
1) COVER UP the thing you want to find and WRITE DOWN what's left.
2) Now PUT IN THE VALUES for the other two things and WORK IT OUT.

E.g. to get the formula for speed from the triangle, cover up S and you're left with $\frac{D}{T}$.

EXAMPLES:

1. Rob cycles 18 miles in 2 hours. What is his average speed?

1) You want speed so covering S gives: $S = \frac{D}{T}$

2) Put in the numbers — and don't forget the units. $S = 18 \div 2$
$= 9$ mph

CHECK YOUR UNITS MATCH
If the distance is in miles and the time is in hours then you'll get a speed in mph.

2. A cheetah runs at a constant speed of 27 m/s for 20 s. What distance does it cover?

1) You want distance so covering D gives: $D = S \times T$

2) Put in the numbers — and don't forget the units. $D = 27 \times 20$
$= 540$ m

UNITS CHECK: m/s and s go in so m comes out.

You can Factorise Quadratic Equations

(Grade 5)

1) You can solve quadratic equations by factorising.
2) 'Factorising a quadratic' means 'putting it into 2 brackets'.

Factorising Quadratics

1) ALWAYS rearrange into the STANDARD FORMAT: $x^2 + bx + c = 0$.
2) Write down the TWO BRACKETS with the x's in: $(x \quad)(x \quad) = 0$.
3) Then find 2 numbers that MULTIPLY to give 'c' (the number term) but also ADD/SUBTRACT to give 'b' (the number in front of the x term).
4) Fill in the +/– signs and make sure they work out properly.
5) As an ESSENTIAL CHECK, EXPAND the brackets to make sure they give the original equation.

Ignore any minus signs at this stage.

3) As well as factorising a quadratic, you might be asked to solve the equation. This just means finding the values of x that make each bracket 0 (see example below).

EXAMPLE: Solve $x^2 - x = 12$.

1) $x^2 - x - 12 = 0$
$(b = -1, c = -12)$

2) $(x \quad)(x \quad) = 0$

3)
1 × 12 Add/subtract to give: 13 or 11
2 × 6 Add/subtract to give: 8 or 4
3 × 4 Add/subtract to give: 7 or 1

This is what we want.

$(x \quad 3)(x \quad 4) = 0$

4) $(x + 3)(x - 4) = 0$

5) Check:
$(x + 3)(x - 4) = x^2 - 4x + 3x - 12$
$= x^2 - x - 12$ ✓

6) $(x + 3) = 0 \Rightarrow x = -3$
$(x - 4) = 0 \Rightarrow x = 4$

1) Rearrange into the standard format.
2) Write down two brackets with x's in.
3) Find the right pair of numbers that multiply to give c (= 12), and add or subtract to give b (= 1) (remember, we're ignoring the +/– signs for now).
4) Now fill in the +/– signs so that 3 and 4 add/subtract to give –1 (= b).
5) ESSENTIAL check — EXPAND the brackets to make sure they give the original expression.

But we're not finished yet — we've only factorised it, we still need to...

6) SOLVE THE EQUATION by setting each bracket equal to 0.

To help you work out which signs you need, look at c.
- If c is positive, the signs will be the same — both positive or both negative.
- If c is negative the signs will be different — one positive and one negative.

Standard Form

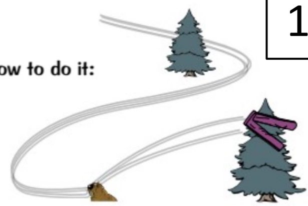
11

Standard form is useful for writing VERY BIG or VERY SMALL numbers in a more convenient way.

E.g. 56 000 000 000 would be 5.6×10^{10} in standard form.

0.000 000 003 45 would be 3.45×10^{-9} in standard form.

But ANY NUMBER can be written in standard form and you need to know how to do it:



What it Actually is: (Grade 4)

A number written in standard form must always be in exactly this form:

$$A \times 10^n$$

This number must always be between 1 and 10. (The fancy way of saying this is $1 \leq A < 10$)

This number is just the number of places the decimal point moves.

Learn the Three Rules:

1) The front number must always be between 1 and 10.
2) The power of 10, n, is how far the decimal point moves.
3) n is positive for BIG numbers, n is negative for SMALL numbers. (This is much better than rules based on which way the decimal point moves.)

Four Important Examples: (Grade 4)

1 Express 35 600 in standard form.

1) Move the decimal point until 35 600 becomes 3.56 ($1 \leq A < 10$)
2) The decimal point has moved 4 places so n = 4, giving: 10^4
3) 35 600 is a big number so n is +4, not –4

3 5 6 0 0.0
$= 3.56 \times 10^4$

2 Express 0.0000623 in standard form.

1) The decimal point must move 5 places to give 6.23 ($1 \leq A < 10$). So the power of 10 is 5.
2) Since 0.0000623 is a small number it must be 10^{-5} not 10^{+5}

0.0 0 0 0 6 2 3
$= 6.23 \times 10^{-5}$

3 Express 4.95×10^{-3} as an ordinary number.

1) The power of 10 is negative, so it's a small number — the answer will be less than 1.
2) The power is –3, so the decimal point moves 3 places.

0 0 0 4.9 5 $\times 10^{-3}$
$= 0.00495$

4 Which is the largest number in the following list? 9.5×10^8 2.7×10^5 3.6×10^8 5.6×10^6

1) Compare the powers first. — 9.5×10^8 and 3.6×10^8 have the biggest powers so one of them is the largest.
2) Then, compare the front numbers. — 9.5 is greater than 3.6. So 9.5×10^8 is the largest number.

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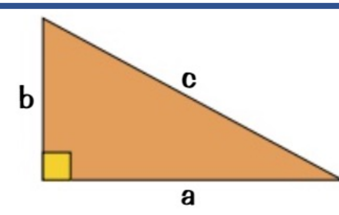
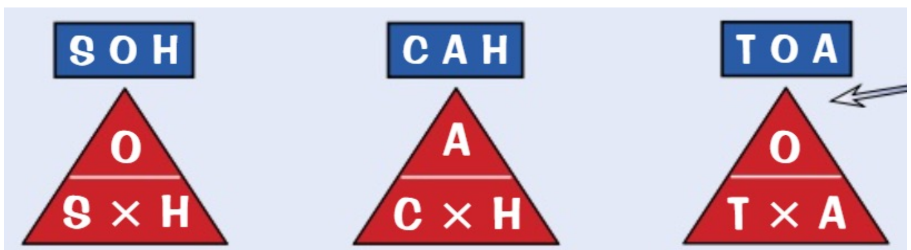
Use trial and error to find how many years it will be before the value drops below £3000. — If $n = 8$, $6500(0.91)^8 = 3056.6414...$; $n = 9$, $6500(0.91)^9 = 2781.5437...$

It will be 9 years before the car is worth less than £3000.

$$a^2 + b^2 = c^2$$





How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

Year 11 Mathematics (Foundation): Low Stake Test scores (Autumn)

Tudor Values
Character Habits
Learning Habits

Topics	Date	Score
Pythagoras' theorem, sharing using ratio, four operations with fractions, Proportion (Recipes), Best buy, Exchange Rates, Simultaneous Equations and Compound Measures (Speed).		
Trigonometry, Tree diagrams, angles on parallel lines, Reverse percentages. Compound interest, Standard form and Plotting linear graphs.		
Forming and solving equations, Sequences (Nth Term), Estimated Mean, Expanding double brackets, Factorising Quadratic Expressions and Area and Circumference of Circles.		
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