Knowledge Organiser: Year 11 (H)

1

Estimating Calculations

1. Round everything off to 1 significant figure.
2. Then work out the answer using these nice easy numbers.
3. Show all your working or you won't get the marks.

Have a look at the previous page to remind yourself how to round to 1 s.f.

EXAMPLES:

1. Estimate the value of 42.6 × 12.1.

1) Round each number to 1 s.f.
2) Do the calculation with the rounded numbers.

$42.6 \times 12.1 \approx 40 \times 10$

$= 400$

≈ means 'approximately equal to'.

You might have to say if it's an underestimate or an overestimate. Here, you rounded both numbers down, so it's an underestimate.

2. Estimate the value of $\frac{\sqrt{6242 \div 57}}{9.8 - 4.7}$.

Don't be put off by the square root, just round each number to 1 s.f. and do the calculation.

$$\frac{\sqrt{6242 \div 57}}{9.8 - 4.7} \approx \frac{\sqrt{6000 \div 60}}{10 - 5} = \frac{\sqrt{100}}{5} = \frac{10}{5} = 2$$

3. Jo has a cake-making business. She spent £984.69 on flour last year. A bag of flour costs £1.89, and she makes an average of 5 cakes from each bag of flour. Work out an estimate of how many cakes she made last year.

1) Estimate number of bags of flour — round numbers to 1 s.f.

$$\text{Number of bags of flour} = \frac{984.69}{1.89} \approx \frac{1000}{2} = 500$$

2) Multiply to find the number of cakes.

$$\text{Number of cakes} \approx 500 \times 5 = 2500$$

Don't panic if you get a 'real-life' estimating question — just round everything to 1 s.f. as before.

2

Algebraic Fractions

Unfortunately, fractions aren't limited to numbers — you can get algebraic fractions too. Fortunately, everything you learnt about fractions on p.5-6 can be applied to algebraic fractions as well.

Simplifying Algebraic Fractions

You can simplify algebraic fractions by cancelling terms on the top and bottom — just deal with each letter individually and cancel as much as you can. You might have to factorise first (see pages 19 and 25-26).

EXAMPLES:

1. Simplify $\frac{21x^3y^2}{14xy^3}$

÷7 on the top and bottom

÷x on the top and bottom to leave x^2 on the top

÷y^2 on the top and bottom to leave y on the bottom

$$\frac{\overset{3}{\cancel{21}}\overset{x^2}{\cancel{x^3}}\cancel{y^2}}{\underset{2}{\cancel{14}}\cancel{x}\underset{y}{\cancel{y^3}}} = \frac{3x^2}{2y}$$

2. Simplify $\frac{x^2 - 16}{x^2 + 2x - 8}$

Factorise the top using D.O.T.S.

$$\frac{\cancel{(x+4)}(x-4)}{(x-2)\cancel{(x+4)}} = \frac{x-4}{x-2}$$

Factorise the quadratic on the bottom

Then cancel the common factor of $(x + 4)$

3

Fractions without a Calculator

3) Multiplying

Multiply top and bottom separately. Then simplify your fraction as far as possible.

EXAMPLE: Find $\frac{8}{5} \times \frac{7}{12}$.

Multiply the top and bottom separately: $\frac{8}{5} \times \frac{7}{12} = \frac{8 \times 7}{5 \times 12}$

Then simplify — top and bottom both divide by 4. $= \frac{56}{60} = \frac{14}{15}$

4) Dividing

Turn the 2nd fraction UPSIDE DOWN and then multiply:

When you're multiplying or dividing with mixed numbers, always turn them into improper fractions first.

EXAMPLE: Find $2\frac{1}{3} \div 3\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{3} \div 3\frac{1}{2} = \frac{7}{3} \div \frac{7}{2}$

Turn $\frac{7}{2}$ upside down and multiply: $= \frac{7}{3} \times \frac{2}{7} = \frac{7 \times 2}{3 \times 7}$

Simplify — top and bottom both divide by 7. $= \frac{14}{21} = \frac{2}{3}$

6) Adding, subtracting — sort the denominators first

1) Make sure the denominators are the same (see above).
2) Add (or subtract) the top lines only.

If you're adding or subtracting mixed numbers, it usually helps to convert them to improper fractions first.

EXAMPLE: Calculate $2\frac{1}{5} - 1\frac{1}{2}$.

Rewrite the mixed numbers as improper fractions: $2\frac{1}{5} - 1\frac{1}{2} = \frac{11}{5} - \frac{3}{2}$

Find a common denominator: $= \frac{22}{10} - \frac{15}{10}$

Combine the top lines: $= \frac{22 - 15}{10} = \frac{7}{10}$

4

The Three Tricky Rules:

8) NEGATIVE Powers — Turn it Upside-Down

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

e.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $a^{-4} = \frac{1}{a^4}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{+2} = \frac{5^2}{3^2} = \frac{25}{9}$

9) FRACTIONAL POWERS

The power $\frac{1}{2}$ means Square Root,
The power $\frac{1}{3}$ means Cube Root,
The power $\frac{1}{4}$ means Fourth Root etc.

e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$
$z^{\frac{1}{5}} = \sqrt[5]{z}$

The one to really watch is when you get a negative fraction like $49^{-1/2}$ — people get mixed up and think that the minus is the square root, and forget to turn it upside down as well.

10) TWO-STAGE FRACTIONAL POWERS

With fractional powers like $64^{\frac{5}{6}}$ always split the fraction into a root and a power, and do them in that order: root first, then power: $(64)^{\frac{1}{6} \times 5} = (64^{\frac{1}{6}})^5 = (2)^5 = 32$.

EXAMPLE: Simplify $(3a^2b^4c)^3$

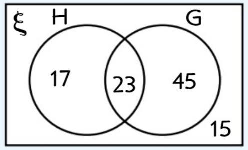
Just deal with each bit separately:

$$= (3)^3 \times (a^2)^3 \times (b^4)^3 \times (c)^3$$
$$= 27 \times a^{2\times3} \times b^{4\times3} \times c^3$$
$$= 27a^6b^{12}c^3$$

You simplify algebraic fractions using the power rules (though you might not realise it). So if you had to simplify e.g. $\frac{p^3q^6}{p^2q^3}$, you'd just cancel using the power rules to get $p^{3-2}q^{6-3} = pq^3$.

Finding Probabilities from Venn Diagrams

EXAMPLE: The Venn diagram on the right shows the number of Year 10 pupils going on the History (H) and Geography (G) school trips.



Find the probability that a randomly selected Year 10 pupil is:

a) not going on the History trip.

n(Year 10 pupils) = 17 + 23 + 45 + 15 = 100
n(Not going on History trip) = 45 + 15 = 60
P(Not going on History trip) = $\frac{60}{100} = \frac{3}{5} = 0.6$

Use the formula from p.106 to find the probabilities.

b) not going on the History trip but going on the Geography trip.

n(Not going on History trip but going on Geography trip) = 45
P(Not going on History trip but going on Geography trip) = $\frac{45}{100} = \frac{9}{20} = 0.45$

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c) going on the Geography trip given that they're not going on the History trip.

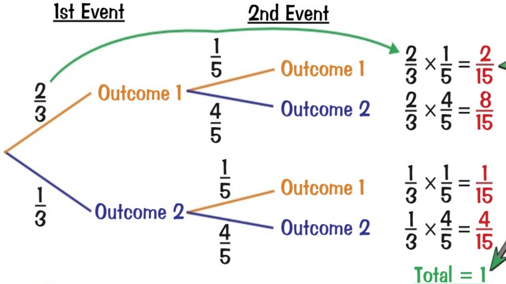
Careful here — think of this as selecting a pupil going on the Geography trip from those not going on the History trip.

P(Going on Geography trip given not going on History trip) = $\frac{45}{45 + 15}$
$= \frac{45}{60} = \frac{3}{4} = 0.75$

You could also use the conditional probability formula and your answers to parts a) and b).

Tree Diagrams

On any set of branches which meet at a point, the probabilities must add up to 1.



2) Multiply along the branches to get the end probabilities.

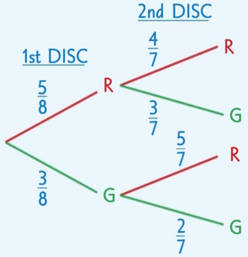
3) Check your diagram — the end probabilities must add up to 1.

4) To answer any question, add up the relevant end probabilities (see below).

A good way to deal with conditional probability questions is to draw a tree diagram. The probabilities on a set of branches will change depending on the previous event.

This example was done 'with replacement' on p.111.

EXAMPLE: A box contains 5 red discs and 3 green discs. Two discs are taken at random without replacement. Find the probability that both discs are the same colour.



The probabilities for the 2nd pick depend on the colour of the 1st disc picked. This is because the 1st disc is not replaced.

P(both discs are red) = P(R and R) = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

P(both discs are green) = P(G and G) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

P(both discs are same colour) = P(R and R or G and G)
$= \frac{20}{56} + \frac{6}{56} = \frac{26}{56} = \frac{13}{28}$

6

Rearranging Formulas

...the Subject Appears Twice

7

Go home and cry. No, not really — you'll just have to do some factorising, usually in step 5.

EXAMPLE: Make p the subject of the formula $q = \frac{p+1}{p-1}$.

There aren't any square roots so ignore step 1.

2) Get rid of any fractions. $q(p - 1) = p + 1$

3) Multiply out any brackets. $pq - q = p + 1$

4) Collect all the subject terms on one side and all non-subject terms on the other.
$pq - p = q + 1$

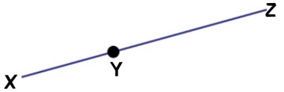
5) Combine like terms on each side of the equation. $p(q - 1) = q + 1$

This is where you factorise — p was in both terms on the LHS so it comes out as a common factor.

6) Divide both sides by (q – 1) to give 'p = '. $p = \frac{q+1}{q-1}$ (p isn't squared, so you don't need step 7.)

Vectors Along a Straight Line

1) You can use vectors to show that points lie on a straight line.
2) You need to show that the vectors along each part of the line point in the same direction — i.e. they're scalar multiples of each other.



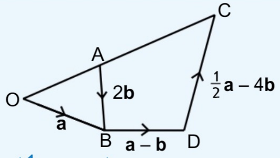
If XYZ is a straight line then $\overrightarrow{XY}$ must be a scalar multiple of $\overrightarrow{YZ}$.

EXAMPLE:

In the diagram,

$\overrightarrow{OB} = \mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BD} = \mathbf{a} - \mathbf{b}$ and $\overrightarrow{DC} = \frac{1}{2}\mathbf{a} - 4\mathbf{b}$.

Show that OAC is a straight line.



1) Work out the vectors along the two parts of OAC (OA and AC) using the vectors you know.

$\overrightarrow{OA} = \underset{\sim}{a} - 2\underset{\sim}{b}$

$\overrightarrow{AC} = 2\underset{\sim}{b} + (\underset{\sim}{a} - \underset{\sim}{b}) + \left(\frac{1}{2}\underset{\sim}{a} - 4\underset{\sim}{b}\right)$

$= \frac{3}{2}\underset{\sim}{a} - 3\underset{\sim}{b} = \frac{3}{2}(\underset{\sim}{a} - 2\underset{\sim}{b})$

2) Check that $\overrightarrow{AC}$ is a scalar multiple of $\overrightarrow{OA}$.

So, $\overrightarrow{AC} = \frac{3}{2}\overrightarrow{OA}$.

3) Explain why this means OAC is a straight line.

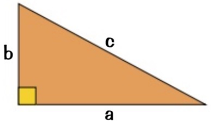
$\overrightarrow{AC}$ is a scalar multiple of $\overrightarrow{OA}$, so OAC must be a straight line.

Pythagoras' Theorem

Pythagoras' theorem sounds hard but it's actually dead simple.
It's also dead important, so make sure you really get your teeth into it.

Pythagoras' Theorem — $a^2 + b^2 = c^2$

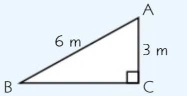
1) PYTHAGORAS' THEOREM only works for RIGHT-ANGLED TRIANGLES.
2) Pythagoras uses two sides to find the third side.
3) The BASIC FORMULA for Pythagoras is $a^2 + b^2 = c^2$
4) Make sure you get the numbers in the RIGHT PLACE. c is the longest side (called the hypotenuse) and it's always opposite the right angle.
5) Always CHECK that your answer is SENSIBLE.



$a^2 + b^2 = c^2$

EXAMPLE:

ABC is a right-angled triangle.
AB = 6 m and AC = 3 m.
Find the exact length of BC.



It's not always c you need to find — loads of people go wrong here.

1) Write down the formula. — $a^2 + b^2 = c^2$
2) Put in the numbers. — $BC^2 + 3^2 = 6^2$
3) Rearrange the equation. — $BC^2 = 6^2 - 3^2 = 36 - 9 = 27$
4) Take square roots to find BC. — $BC = \sqrt{27} = 3\sqrt{3}$ m
5) 'Exact length' means you should give your answer as a surd — simplified if possible.

Remember to check the answer's sensible — here it's about 5.2, which is between 3 and 6, so that seems about right...

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y = mx + c

Using 'y = mx + c' is the most straightforward way of dealing with straight-line equations, and it's very useful in exams. The first thing you have to do though is rearrange the equation into the standard format like this:

Straight line:		Rearranged into 'y = mx +c'	
$y = 2 + 3x$	→	$y = 3x + 2$	$(m = 3, c = 2)$
$x - y = 0$	→	$y = x + 0$	$(m = 1, c = 0)$
$4x - 3 = 5y$	→	$y = \frac{4}{5}x - \frac{3}{5}$	$(m = \frac{4}{5}, c = -\frac{3}{5})$

where:
'm' = gradient of the line.
'c' = 'y-intercept' (where it hits the y-axis)

WATCH OUT: people mix up 'm' and 'c' when they get something like y = 5 + 2x. Remember, 'm' is the number in front of the 'x' and 'c' is the number on its own.

Finding the Equation of a Straight-Line Graph

When you're given the graph itself, it's quick and easy to find the equation of the straight line.

EXAMPLE:

Find the equation of the line on the graph in the form $y = mx + c$.

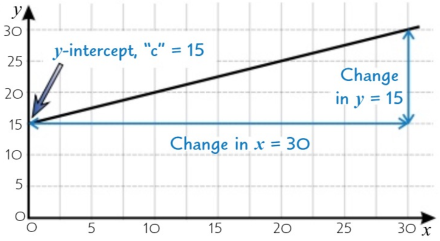
1) Find 'm' (gradient) and 'c' (y-intercept).

$$'m' = \frac{\text{change in } y}{\text{change in } x} = \frac{15}{30} = \frac{1}{2}$$

'c' = 15

2) Use these to write the equation in the form y = mx + c.

$$y = \frac{1}{2}x + 15$$

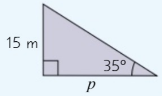


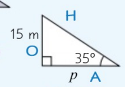
Trigonometry — Examples

Here are some lovely examples using the method from p.96 to help you through the trials of trig.

Examples:

1 Find the length of p in the triangle shown to 3 s.f.





1) Label the sides
2) Write down — SOH CAH TOA
3) O and A involved
4) Write down the formula triangle

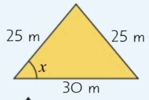
5) You want A so cover it up to give — $A = \frac{O}{T}$
6) Put in the numbers — $p = \frac{15}{\tan 35°} = 21.422...$

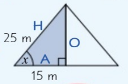
= 21.4 m (3 s.f.)

Is it sensible? Yes, it's a bit bigger than 15, as the diagram suggests.

2 Find the angle x in this triangle to 1 d.p.

It's an isosceles triangle so split it down the middle to get a right-angled triangle.





1) Label the sides
2) Write down — SOH CAH TOA
3) A and H involved
4) Write down the formula triangle

5) You want the angle so cover up C to give — $C = \frac{A}{H}$
6) Put in the numbers — $\cos x = \frac{15}{25} = 0.6$
7) Find the inverse — $\Rightarrow x = \cos^{-1}(0.6) = 53.1301...°$

= 53.1° (1 d.p.)

Is it sensible? Yes, the angle looks about 50°.

Where There's a Sign Change, There's a Solution

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If you're trying to solve an equation that equals 0, there's one very important thing to remember:

> If there's a sign change (i.e. from positive to negative or vice versa) when you put two numbers into the equation, there's a solution between those numbers.

Think about the equation $x^3 - 3x - 1 = 0$. When $x = -1$, the expression gives $(-1)^3 - 3(-1) - 1 = 1$, which is positive, and when $x = -2$ the expression gives $(-2)^3 - 3(-2) - 1 = -3$, which is negative. This means that the expression will be 0 for some value between $x = -1$ and $x = -2$ (the solution).

Use Iteration When an Equation is Too Hard to Solve

Not all equations can be solved using the methods you've seen so far in this section (e.g. factorising, the quadratic formula etc.). But if you know an interval that contains a solution to an equation, you can use an iterative method to find the approximate value of the solution.

EXAMPLE: Use the iteration machine below to find a solution to the equation $x^3 - 3x - 1 = 0$ to 1 d.p. Use the starting value $x_0 = -1$.

Look back at p.32 for more on the x_n notation.

1. Start with x_n →
2. Find the value of x_{n+1} by using the formula $x_{n+1} = \sqrt[3]{1 + 3x_n}$. →
3. If $x_n = x_{n+1}$ rounded to 1 d.p. then stop. If $x_n \neq x_{n+1}$ rounded to 1 d.p. go back to step 1 and repeat using x_{n+1}.

Put the value of x_0 into the iteration machine:

$x_0 = -1$ $\quad$ $x_1 = -1.25992... \neq x_0$ to 1 d.p.

$x_2 = -1.40605... \neq x_1$ to 1 d.p. $\quad$ $x_3 = -1.47639... \neq x_2$ to 1 d.p.

$x_4 = -1.50798... = x_3$ to 1 d.p.

x_3 and x_4 both round to –1.5 to 1 d.p. so the solution is $x = -1.5$ to 1 d.p.

This is the same example as above so the solution is the same.

Quadratic Formula — Five Crucial Details

1) Take it nice and slowly — always write it down in stages as you go.
2) **WHENEVER YOU GET A MINUS SIGN, THE ALARM BELLS SHOULD ALWAYS RING!**
3) Remember it's '2a' on the bottom line, not just 'a' — and you divide ALL of the top line by 2a.
4) The ± sign means you end up with two solutions (by replacing it in the final step with '+' and '–').
5) If you get a negative number inside your square root, go back and check your working. Some quadratics do have a negative value in the square root, but they won't come up at GCSE.

If either 'a' or 'c' is negative, the -4ac effectively becomes +4ac, so watch out. Also, be careful if b is negative, as -b will be positive.

EXAMPLE: Solve $3x^2 + 7x = 1$, giving your answers to 2 decimal places.

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$$3x^2 + 7x - 1 = 0$$

$a = 3, \quad b = 7, \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times -1}}{2 \times 3}$$
$$= \frac{-7 \pm \sqrt{49 + 12}}{6}$$
$$= \frac{-7 \pm \sqrt{61}}{6}$$
$$= \frac{-7 + \sqrt{61}}{6} \text{ or } \frac{-7 - \sqrt{61}}{6}$$
$$= 0.1350... \text{ or } -2.468...$$

So to 2 d.p. the solutions are: $x = 0.14$ or -2.47

Notice that you do two calculations at the final stage — one + and one –.

1) First get it into the form $ax^2 + bx + c = 0$.
2) Then carefully identify a, b and c.
3) Put these values into the quadratic formula and write down each stage.
4) Finally, as a check put these values back into the original equation: E.g. for x = 0.1350: $3 \times 0.135^2 + 7 \times 0.135 = 0.999675$, which is 1, as near as...

When to use the quadratic formula:
- If you have a quadratic that won't easily factorise.
- If the question mentions decimal places or significant figures.

If the question asks for exact answers or surds (though this could be completing the square instead — see next page).

Finding the nth Term of a Quadratic Sequence

A quadratic sequence has an n^2 term — the difference between the terms changes as you go through the sequence, but the difference between the differences is the same each time.

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EXAMPLE: Find an expression for the nth term of the sequence that starts 10, 14, 20, 28...

n: 1 2 3 4

term: 10 14 20 28

+4 +6 +8

+2 +2

So the expression will contain an n^2 term.

term:	10	14	20	28
n^2:	1	4	9	16
term – n^2:	9	10	11	12

The expression for this linear sequence is n + 8

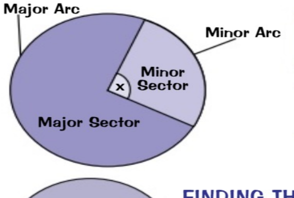
So the expression for the nth term is $n^2 + n + 8$

1) Find the difference between each pair of terms.
2) The difference is changing, so work out the difference between the differences.
3) Divide this value by 2 — this gives the coefficient of the n^2 term (here it's 2 ÷ 2 = 1).
4) Subtract the n^2 term from each term in the sequence. This will give you a linear sequence.
5) Find the rule for the nth term of the linear sequence (see above) and add this on to the n^2 term.

Again, make sure you check your expression by putting the first few values of n back in — so n = 1 gives $1^2 + 1 + 8 = 10$, n = 2 gives $2^2 + 2 + 8 = 14$ and so on.

Areas of Sectors and Segments

These next ones are a bit more tricky — before you try and learn the formulas, make sure you know what a sector, an arc and a segment are (I've helpfully labelled the diagrams below — I'm nice like that).

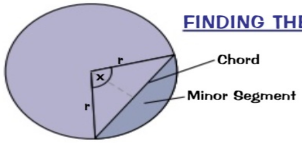


$$\text{Area of Sector} = \frac{x}{360} \times \text{Area of full Circle}$$

(Pretty obvious really, isn't it?)

$$\text{Length of Arc} = \frac{x}{360} \times \text{Circumference of full Circle}$$

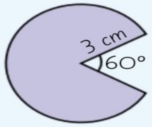
(Obvious again, no?)



FINDING THE AREA OF A SEGMENT is OK if you know the formulas.

1) Find the area of the sector using the above formula.
2) Find the area of the triangle, then subtract it from the sector's area. You can do this using the '½ ab sin C' formula for the area of the triangle (see previous page), which becomes: ½ r^2sin x.

EXAMPLE: In the diagram on the right, a sector with angle 60° has been cut out of a circle with radius 3 cm. Find the exact area of the shaded shape.



First find the angle of the shaded sector (this is the major sector):

$360° - 60° = 300°$

Then use the formula to find the area of the shaded sector:

$$\text{area of sector} = \frac{x}{360} \times \pi r^2 = \frac{300}{360} \times \pi \times 3^2$$
$$= \frac{5}{6} \times \pi \times 9 = \frac{15}{2}\pi \text{ cm}^2$$

'Exact area' means leave your answer in terms of π.

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How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice**: Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

Year 11 Mathematics (Higher): Low Stake Test scores (Autumn)

Tudor Values
Character Habits
Learning Habits

Topics	Date	Score
Mixed numbers (4 operations), Estimation, Negative Indices, Fractional Indices, Tree diagrams and Simplifying algebraic fractions.		
Venn diagrams, Vectors, Iteration, Simultaneous linear equations and the equation of a line.		
Pythagoras, Trigonometry, Area of a segment, Fractional equations, changing the subject of a formula, Quadratic formula and quadratic nth term.		
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