



Knowledge Organiser: Year 11 - Higher (Spring Term)

Find Averages from Grouped Frequency Tables



Add extra columns for 'mid-interval value' and 'frequency \times mid-interval value'.
Add up the values in the 4th column to estimate the **total weight** of the 60 children.

$$\text{Mean} \approx \frac{\text{total weight}}{\text{number of children}} \leftarrow \begin{array}{l} \text{4th column total} \\ \text{2nd column total} \end{array}$$

$$= \frac{3220}{60} = 53.7 \text{ kg (3 s.f.)}$$

Weight (w kg)	Frequency	Mid-interval value	Frequency \times mid-interval value
$30 < w \leq 40$	8	35	280
$40 < w \leq 50$	16	45	720
$50 < w \leq 60$	18	55	990
$60 < w \leq 70$	12	65	780
$70 < w \leq 80$	6	75	450
Total	60	—	3220

2

Box Plots show the Spread of a Data Set

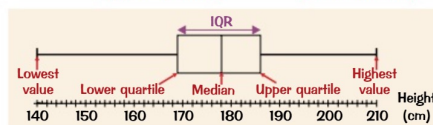


- 1) The **lower quartile** Q_1 , the **median** Q_2 and the **upper quartile** Q_3 are the values **25%** ($\frac{1}{4}$), **50%** ($\frac{1}{2}$) and **75%** ($\frac{3}{4}$) of the way through an ordered set of data. So if a set of data has n values, you can work out the **positions** of the **quartiles** using these formulas:

$$Q_1: (n + 1)/4 \quad Q_2: (n + 1)/2 \quad Q_3: 3(n + 1)/4$$

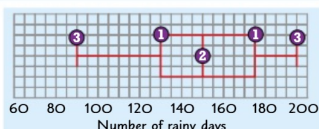
- 2) The **INTERQUARTILE RANGE (IQR)** is the **difference** between the **upper quartile** and the **lower quartile** and contains the **middle 50%** of values.

- 3) A **box plot** shows the **minimum** and **maximum** values in a data set and the values of the **quartiles**. But it **doesn't** tell you the **individual** data values.



EXAMPLE:

This table gives information about the numbers of rainy days last year in some cities. On the grid below, draw a box plot to show the information.



- 1) Mark on the **quartiles** and **draw the box**.
- 2) Draw a **line** at the **median**.
- 3) Mark on the **minimum** and **maximum** points and **join them to the box** with horizontal lines.

Minimum number	90
Maximum number	195
Lower quartile	130
Median	150
Upper quartile	175

3

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cumulative Frequency

Cumulative frequency just means **adding it up as you go along** — i.e. the **total frequency so far**. You need to be able to **draw a cumulative frequency graph** and **make estimates** from it.



EXAMPLE:

The table below shows information about the heights of a group of people.

- a) Draw a cumulative frequency graph for the data.
- b) Use your graph to **estimate** the **median** and **interquartile range** of the heights.

Height (h cm)	Frequency	Cumulative Frequency
$140 < h \leq 150$	4	4
$150 < h \leq 160$	9	$4 + 9 = 13$
$160 < h \leq 170$	20	$13 + 20 = 33$
$170 < h \leq 180$	33	$33 + 33 = 66$
$180 < h \leq 190$	36	$66 + 36 = 102$
$190 < h \leq 200$	15	$102 + 15 = 117$
$200 < h \leq 210$	3	$117 + 3 = 120$

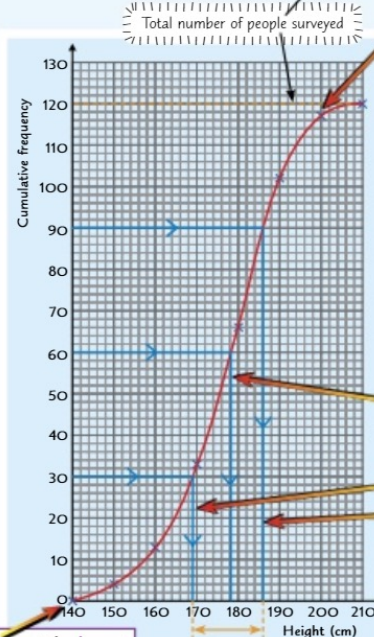
To Draw the Graph...

- 1) Add a 'CUMULATIVE FREQUENCY' COLUMN to the table — and fill it in with the **RUNNING TOTAL** of the **frequency column**.
- 2) **PLOT** points using the **HIGHEST VALUE** in each class and the **CUMULATIVE FREQUENCY**. (150, 4), (160, 13), etc.
- 3) **Join** the points with a **smooth curve** or straight lines.

To Find the Vital Statistics...

- 1) **MEDIAN** — go **halfway up** the side, **across** to the **curve**, then **down** and read off the **bottom scale**.
- 2) **LOWER AND UPPER QUARTILES** — go $\frac{1}{4}$ and $\frac{3}{4}$ up the side, **across** to the **curve**, then **down** and read off the **bottom scale**.
- 3) **INTERQUARTILE RANGE** — the **distance** between the lower and upper quartiles.

1



Plot zero at the lowest value in the first class.

Interquartile range

- 1) The halfway point is at $\frac{1}{2} \times 120 = 60$. Reading across and down gives a **median of 178 cm**.
- 2) $\frac{1}{4}$ of the way up is at $\frac{1}{4} \times 120 = 30$. Reading across and down gives a lower quartile of 169 cm.
- 3) $\frac{3}{4}$ of the way up is at $\frac{3}{4} \times 120 = 90$. Reading across and down gives an upper quartile of 186 cm.

More Estimating...

To use the graph to **estimate** the **number** of values that are **less than** or **greater than** a given value: Go **along** the bottom scale to the **given value**, **up** to the **curve**, then **across** to the **cumulative frequency**. (See the **question below** for an **example**.)

The values you **read** from the graph are **estimates** because they're based on **grouped** data — you don't know how the **actual** data values are **spread** within each class.

SOH



CAH



TOA



Histograms Show Frequency Density

- 1) The vertical axis on a histogram is always called frequency density. You work it out using this formula:

$$\text{Frequency Density} = \text{Frequency} \div \text{Class Width}$$

Remember... 'frequency' is just another way of saying 'how much' or 'how many'.

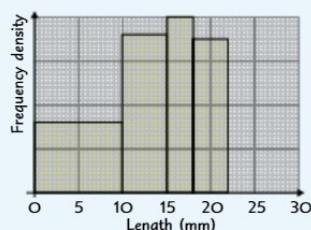
- 2) You can rearrange it to work out how much a bar represents.

$$\text{Frequency} = \text{Frequency Density} \times \text{Class Width} = \text{AREA of bar}$$

EXAMPLE:

This table and histogram show the lengths of beetles found in a garden.

Length (mm)	Frequency
$0 < x \leq 10$	32
$10 < x \leq 15$	36
$15 < x \leq 18$	
$18 < x \leq 22$	28
$22 < x \leq 30$	16

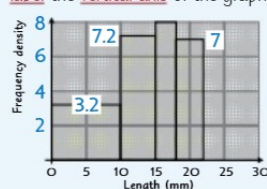


- a) Use the histogram to find the missing entry in the table.

- 1) Add a frequency density column to the table and fill in what you can using the formula.

Frequency density
$32 \div 10 = 3.2$
$36 \div 5 = 7.2$
$28 \div 4 = 7$
$16 \div 8 = 2$

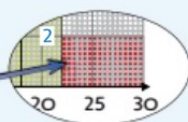
- 2) Use the frequency densities to label the vertical axis of the graph.



- 3) Now use the 3rd bar to find the frequency for the class " $15 < x \leq 18$ ".
Frequency density = 8 and class width = 3.
So frequency = frequency density \times class width = $8 \times 3 = 24$

- b) Use the table to add the bar for the class " $22 < x \leq 30$ " to the histogram.

$$\text{Frequency density} = \text{Frequency} \div \text{Class Width} = \frac{16}{8} = 2$$



- c) Estimate the number of beetles between 7.5 mm and 12.5 mm in length.

Use the formula frequency = frequency density \times class width — multiply the frequency density of the class by the width of the part of that class you're interested in.

$$\begin{aligned} & 3.2 \times (10 - 7.5) + 7.2 \times (12.5 - 10) \\ &= 3.2 \times 2.5 + 7.2 \times 2.5 \\ &= 26 \end{aligned}$$

The Sine and Cosine Rules

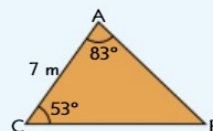
There are four main question types where the sine and cosine rules would be applied. So learn the exact details of these four examples and you'll be laughing. WARNING: if you laugh too much people will think you're crazy.

The Four Examples

1

TWO ANGLES given plus **ANY SIDE** — **SINE RULE** needed.

Find the length of AB for the triangle below.

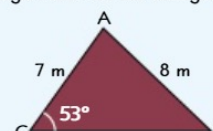


- Don't forget the obvious...
 $B = 180^\circ - 83^\circ - 53^\circ = 44^\circ$
- Put the numbers into the sine rule.
 $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{7}{\sin 44^\circ} = \frac{c}{\sin 53^\circ}$
- Rearrange to find c.
 $\Rightarrow c = \frac{7 \times \sin 53^\circ}{\sin 44^\circ} = 8.05 \text{ m (3 s.f.)}$

2

TWO SIDES given plus an **ANGLE NOT ENCLOSED** by them — **SINE RULE** needed.

Find angle ABC for the triangle shown below.

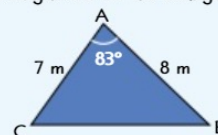


- Put the numbers into the sine rule.
 $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{7}{\sin B} = \frac{8}{\sin 53^\circ}$
- Rearrange to find sin B.
 $\Rightarrow \sin B = \frac{7 \times \sin 53^\circ}{8} = 0.6988...$
- Find the inverse.
 $\Rightarrow B = \sin^{-1}(0.6988...) = 44.3^\circ \text{ (1 d.p.)}$

3

TWO SIDES given plus the **ANGLE ENCLOSED** by them — **COSINE RULE** needed.

Find the length CB for the triangle shown below.



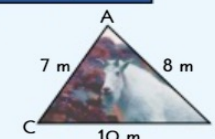
- Put the numbers into the cosine rule.
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $= 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 83^\circ$
 $= 99.3506...$
- Take square roots to find a.
 $a = \sqrt{99.3506...}$
 $= 9.97 \text{ m (3 s.f.)}$

You might come across a triangle that isn't labelled ABC — just relabel it yourself to match the sine and cosine rules.

4

ALL THREE SIDES given but **NO ANGLES** — **COSINE RULE** needed.

Find angle CAB for the triangle shown.



- Use this version of the cosine rule.
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
 $= \frac{49 + 64 - 100}{2 \times 7 \times 8}$
- Put in the numbers.
 $= \frac{13}{112} = 0.11607...$
- Take the inverse to find A.
 $\Rightarrow A = \cos^{-1}(0.11607...) = 83.3^\circ \text{ (1 d.p.)}$

Manipulating Surds — 6 Rules to Learn



There are 6 rules you need to learn for dealing with surds...

1 $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$ — also $(\sqrt{b})^2 = \sqrt{b} \times \sqrt{b} = \sqrt{b \times b} = b$

2 $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ e.g. $\sqrt{\frac{8}{2}} = \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4} = 2$

3 $\sqrt{a} + \sqrt{b}$ — **DO NOTHING** — in other words it is definitely **NOT** $\sqrt{a+b}$

4 $(a + \sqrt{b})^2 = (a + \sqrt{b})(a + \sqrt{b}) = a^2 + 2a\sqrt{b} + b$ — **NOT** just $a^2 + (\sqrt{b})^2$ (see p.18)

5 $(a + \sqrt{b})(a - \sqrt{b}) = a^2 + a\sqrt{b} - a\sqrt{b} - (\sqrt{b})^2 = a^2 - b$ (see p.19).

6 $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ This is known as '**RATIONALISING the denominator**' — it's where you get rid of the $\sqrt{\quad}$ on the bottom of the fraction. For denominators of the form $a \pm \sqrt{b}$, you always multiply by the denominator but **change the sign** in front of the root (see example 3 below).



Use the Rules to Simplify Expressions



EXAMPLES:

1. Write $\sqrt{300} + \sqrt{48} - 2\sqrt{75}$ in the form $a\sqrt{3}$, where a is an integer.

Write each surd in terms of $\sqrt{3}$: $\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

$2\sqrt{75} = 2\sqrt{25 \times 3} = 2 \times \sqrt{25} \times \sqrt{3} = 10\sqrt{3}$

Then do the sum (leaving your answer in terms of $\sqrt{3}$):

$\sqrt{300} + \sqrt{48} - 2\sqrt{75} = 10\sqrt{3} + 4\sqrt{3} - 10\sqrt{3} = 4\sqrt{3}$

2. A rectangle with length $4x$ cm and width x cm has an area of 32 cm². Find the exact value of x , giving your answer in its simplest form.

Area of rectangle = length \times width = $4x \times x = 4x^2$

So $4x^2 = 32$

$x^2 = 8$

$x = \pm\sqrt{8}$

You can ignore the negative square root (see p.22) as length must be positive.

'Exact value' means you have to leave your answer in surd form, so get $\sqrt{8}$ into its simplest form:

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \sqrt{2}$

$= 2\sqrt{2}$ So $x = 2\sqrt{2}$

3. Write $\frac{3}{2+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

To **rationalise the denominator**, multiply top and bottom by $2 - \sqrt{5}$:

$$\begin{aligned} \frac{3}{2+\sqrt{5}} &= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} \\ &= \frac{6-3\sqrt{5}}{2^2 - 2\sqrt{5} + 2\sqrt{5} - (\sqrt{5})^2} \\ &= \frac{6-3\sqrt{5}}{4-5} = \frac{6-3\sqrt{5}}{-1} = -6 + 3\sqrt{5} \end{aligned}$$

(so $a = -6$ and $b = 3$)



6

Compound Growth and Decay

One more sneaky % type for you... Unlike **simple interest**, in **compound interest** the amount added on (or taken away) **changes** each time — it's a percentage of the **new amount**, rather than the **original amount**.

The Formula



This topic is simple if you **LEARN THIS FORMULA**. If you don't, it's pretty well impossible:

$$\text{Amount after } n \text{ days/hours/years} \rightarrow N = N_0 \times (\text{multiplier})^n \leftarrow \text{Number of days/hours/years}$$

Initial amount $\rightarrow N_0$ \rightarrow Percentage change multiplier

E.g. 5% increase is 1.05 ($= 1 + 0.05$)
26% decrease is 0.74 ($= 1 - 0.26$)

3 Examples to show you how EASY it is:



Compound interest is a popular context for these questions — it means the interest is **added on each time**, and the next lot of interest is calculated using the **new total** rather than the original amount.

EXAMPLE:

Daniel invests £1000 in a savings account which pays 8% compound interest per annum. How much will there be after 6 years?

Use the **formula**: $\text{Amount} = 1000(1.08)^6 = \text{£}1586.87$

initial amount \rightarrow 8% increase \rightarrow 6 years

'Per annum' just means 'each year'.

Depreciation questions are about things (e.g. cars) which **decrease in value** over time.

EXAMPLE:

Susan has just bought a car for £6500.

a) If the car depreciates by 9% each year, how much will it be worth in 3 years' time?

Use the **formula**: $\text{Value} = 6500(0.91)^3 = \text{£}4898.21$

b) How many complete years will it be before the car is worth less than £3000?

Use the **formula** again but this time you know don't know n .

$\text{Value} = 6500(0.91)^n$

If $n = 8$, $6500(0.91)^8 = 3056.6414...$

$n = 9$, $6500(0.91)^9 = 2781.5437...$

Use **trial and error** to find how many years it will be before the value drops below £3000.

It will be **9 years** before the car is worth less than £3000.

7

Inverse Functions



The **inverse** of a function $f(x)$ is another function, $f^{-1}(x)$, which **reverses** $f(x)$. Here's the **method** to find it:

- 1) Write out the equation $x = f(y)$
- 2) **Rearrange** the equation to **make y the subject**.
- 3) Finally, **replace** y with $f^{-1}(x)$.

$f(y)$ is just the expression $f(x)$, but with y 's instead of x 's

EXAMPLE:

If $f(x) = \frac{12+x}{3}$, find $f^{-1}(x)$.

So here you just rewrite the function replacing $f(x)$ with x and x with y :

1) Write out $x = f(y)$: $x = \frac{12+y}{3}$

2) Rearrange to make y the subject:

$3x = 12 + y$

$y = 3x - 12$

3) Replace y with $f^{-1}(x)$:

$f^{-1}(x) = 3x - 12$

8

You can check your answer by seeing if $f^{-1}(x)$ does reverse $f(x)$: e.g. $f(9) = \frac{21}{3} = 7$, $f^{-1}(7) = 21 - 12 = 9$

Where There's a Sign Change, There's a Solution

GRADE 7

If you're trying to solve an equation that equals 0, there's one very important thing to remember:

If there's a sign change (i.e. from positive to negative or vice versa) when you put two numbers into the equation, there's a solution between those numbers.

9

Think about the equation $x^2 - 3x - 1 = 0$. When $x = -1$, the expression gives $(-1)^2 - 3(-1) - 1 = 1$, which is positive, and when $x = -2$ the expression gives $(-2)^2 - 3(-2) - 1 = -3$, which is negative. This means that the expression will be 0 for some value between $x = -1$ and $x = -2$ (the solution).

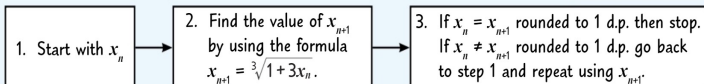
Use Iteration When an Equation is Too Hard to Solve

GRADE 7

Not all equations can be solved using the methods you've seen so far in this section (e.g. factorising, the quadratic formula etc.). But if you know an interval that contains a solution to an equation, you can use an iterative method to find the approximate value of the solution.

EXAMPLE: Use the iteration machine below to find a solution to the equation $x^3 - 3x - 1 = 0$ to 1 d.p. Use the starting value $x_0 = -1$.

Look back at p32 for more on the x_n notation.



Put the value of x_0 into the iteration machine:

$$x_0 = -1 \quad x_1 = -1.25992... \neq x_0 \text{ to 1 d.p.}$$

$$x_2 = -1.40605... \neq x_1 \text{ to 1 d.p.} \quad x_3 = -1.47639... \neq x_2 \text{ to 1 d.p.}$$

$$x_4 = -1.50798... = x_3 \text{ to 1 d.p.}$$

$$x_3 \text{ and } x_4 \text{ both round to } -1.5 \text{ to 1 d.p. so the solution is } x = -1.5 \text{ to 1 d.p.}$$

This is the same example as above so the solution is the same.

Show Things Are Odd, Even or Multiples by Rearranging

Before you get started, there are a few things you need to know — they'll come in very handy when you're trying to prove things.

- Any even number can be written as $2n$ — i.e. $2 \times$ something.
- Any odd number can be written as $2n + 1$ — i.e. $2 \times$ something + 1.
- Consecutive numbers can be written as $n, n + 1, n + 2$ etc. — you can apply this to e.g. consecutive even numbers too (they'd be written as $2n, 2n + 2, 2n + 4$). (In all of these statements, n is just any integer.)
- The sum, difference and product of integers is always an integer.

This can be extended to multiples of other numbers too — e.g. to prove that something is a multiple of 5, show that it can be written as $5 \times$ something.

EXAMPLE: Prove that the sum of any three odd numbers is odd.

Take three odd numbers:
 $2a + 1, 2b + 1$ and $2c + 1$
(they don't have to be consecutive)

Add them together:

$$2a + 1 + 2b + 1 + 2c + 1 = 2a + 2b + 2c + 3 = 2(a + b + c + 1) + 1$$

So the sum of any three odd numbers is odd.

You'll see why I've written 3 as $2 + 1$ in a second.

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EXAMPLE: Prove that $(n + 3)^2 - (n - 2)^2 \equiv 5(2n + 1)$.

Take one side of the equation and play about with it until you get the other side:

$$\begin{aligned} \text{LHS: } (n + 3)^2 - (n - 2)^2 &\equiv n^2 + 6n + 9 - (n^2 - 4n + 4) \\ &\equiv n^2 + 6n + 9 - n^2 + 4n - 4 \\ &\equiv 10n + 5 \\ &\equiv 5(2n + 1) = \text{RHS} \end{aligned}$$

\equiv is the identity symbol, and means that two things are identically equal to each other. So $a + b \equiv b + a$ is true for all values of a and b (unlike an equation, which is only true for certain values).

Percentages

GRADE 4

10

Type 6 — Finding the Original Value

This is the type that most people get wrong — but only because they don't recognise it as this type, and don't apply this simple method:

- Write the amount in the question as a percentage of the original value.
- Divide to find 1% of the original value.
- Multiply by 100 to give the original value ($= 100\%$).

EXAMPLE:

A house increases in value by 10% to £165 000. Find what it was worth before the rise.

Note: The new, not the original value is given.

- An increase of 10% means £165 000 represents 110% of the original value.
- Divide by 110 to find 1% of the original value.
- Then multiply by 100.

$$\begin{aligned} &\times 110 \quad \left\{ \begin{array}{l} £165\,000 = 110\% \\ £1500 = 1\% \end{array} \right. \\ &\times 100 \quad \left\{ \begin{array}{l} £150\,000 = 100\% \end{array} \right. \end{aligned}$$

So the original value was £150 000

If it was a decrease of 10%, then you'd put '£165 000 = 90%' and divide by 90 instead of 110.

Always set them out exactly like this example. The trickiest bit is deciding the top % figure on the right-hand side — the 2nd and 3rd rows are always 1% and 100%.

Adding/Subtracting Algebraic Fractions

GRADE 8

For the common denominator, find something both denominators divide into.

Adding or subtracting is a bit more difficult:

- Work out the common denominator (see p.6).
- Multiply top and bottom of each fraction by whatever gives you the common denominator.
- Add or subtract the numerators only.

Fractions		
$\frac{1}{x} + \frac{1}{3x}$	$\frac{1}{x+1} + \frac{1}{x-2}$	$\frac{1}{x} + \frac{1}{x(x+1)}$
$3x$	$(x+1)(x-2)$	$x(x+1)$
Common denominator		

EXAMPLE:

Write $\frac{3}{(x+3)} + \frac{1}{(x-2)}$ as a single fraction.

$$\begin{aligned} \frac{3}{(x+3)} + \frac{1}{(x-2)} &= \frac{3(x-2)}{(x+3)(x-2)} + \frac{(x+3)}{(x+3)(x-2)} \\ &= \frac{3x-6}{(x+3)(x-2)} + \frac{x+3}{(x+3)(x-2)} = \frac{4x-3}{(x+3)(x-2)} \end{aligned}$$

1st fraction: \times top & bottom by $(x-2)$
2nd fraction: \times top & bottom by $(x+3)$

Add the numerators

Common denominator will be $(x+3)(x-2)$

12

Solving Equations Using Graphs

You can plot graphs to find **approximate solutions** to simultaneous equations or other awkward equations. Plot the equations you want to solve and the solution lies where the lines **intersect**.

Plot Both Graphs and See Where They Cross

EXAMPLE:

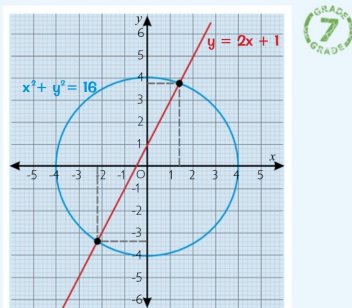
By plotting the graphs, solve the simultaneous equations $x^2 + y^2 = 16$ and $y = 2x + 1$.

1) **DRAW BOTH GRAPHS.**

$x^2 + y^2 = 16$ is the equation of a circle with centre (O, O) and radius 4 (see p.49). Use a pair of compasses to draw it accurately.

2) **LOOK FOR WHERE THE GRAPHS CROSS.**

The straight line crosses the circle at **two points**. Reading the **x** and **y** values of these points gives the solutions $x = 1.4$, $y = 3.8$ and $x = -2.2$, $y = -3.4$ (all to 1 decimal place).



13

Inverse Proportion

4
GRADE

- Two quantities, C and D, are in **inverse proportion** if **increasing** one quantity causes the other quantity to **decrease proportionally**. So if quantity C is **doubled** (or tripled, halved, etc.), quantity D is **halved** (or divided by 3, doubled etc.).
- The rule for finding inverse proportions is:

TIMES for ONE, then DIVIDE for ALL

EXAMPLE:

4 bakers can decorate 100 cakes in 5 hours.

- a) How long would it take 10 bakers to decorate the same number of cakes?

100 cakes will take 1 baker: $5 \times 4 = 20$ hours

So 100 cakes will take 10 bakers: $20 \div 10 = 2$ hours for 10 bakers

- b) How long would it take 11 bakers to decorate 220 cakes?

100 cakes will take 1 baker: 20 hours

1 cake will take 1 baker: $20 \div 100 = 0.2$ hours

220 cakes will take 1 baker: $0.2 \times 220 = 44$ hours

220 cakes will take 11 bakers: $44 \div 11 = 4$ hours

The number of bakers is **inversely proportional** to number of hours — but the number of cakes is **directly proportional** to the number of hours.

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Evaluating Functions

6
GRADE

This is easy — just shove the numbers into the function and you're away.

EXAMPLE:

$f(x) = x^2 - x + 7$. Find a) $f(3)$ and b) $f(-2)$

a) $f(3) = (3)^2 - (3) + 7 = 9 - 3 + 7 = 13$ b) $f(-2) = (-2)^2 - (-2) + 7 = 4 + 2 + 7 = 13$

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Combining Functions

8
GRADE

- You might get a question with **two functions**, e.g. $f(x)$ and $g(x)$, **combined** into a single function (called a **composite function**).
- Composite functions are written e.g. **$fg(x)$** , which means 'do **g** first, then do **f**' — you always do the function **closest** to x first.
- To find a composite function, rewrite $fg(x)$ as **$f(g(x))$** , then replace $g(x)$ with the **expression** it represents and then put this into f .

Watch out — usually $fg(x) \neq gf(x)$. Never assume that they're the same.

EXAMPLE:

If $f(x) = 2x - 10$ and $g(x) = -\frac{x}{2}$, find: a) $fg(x)$ and b) $gf(x)$.

a) $fg(x) = f(g(x)) = f(-\frac{x}{2}) = 2(-\frac{x}{2}) - 10 = -x - 10$

b) $gf(x) = g(f(x)) = g(2x - 10) = -(\frac{2x - 10}{2}) = -(x - 5) = 5 - x$

Direct Proportion

4
GRADE

- Two quantities, A and B, are in **direct proportion** (or just in **proportion**) if increasing one increases the other one **proportionally**. So if quantity A is doubled (or trebled, halved, etc.), so is quantity B.
- Remember this **golden rule** for direct proportion questions:

DIVIDE for ONE, then TIMES for ALL

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EXAMPLE:

Hannah pays £3.60 per 400 g of cheese.

She uses 220 g of cheese to make 4 cheese pasties.

How much would the cheese cost if she wanted to make 50 cheese pasties?

There will often be lots of stages to direct proportion questions — keep track of what you've worked out at each stage.

In 1 **pasty** there is:

$220 \text{ g} \div 4 = 55 \text{ g of cheese}$

So in 50 **pasties** there is:

$55 \text{ g} \times 50 = 2750 \text{ g of cheese}$

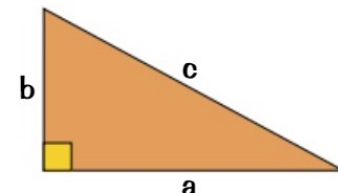
1 **g of cheese** would cost:

$£3.60 \div 400 = 0.9\text{p}$

So 2750 **g of cheese** would cost:

$0.9 \times 2750 = 2475\text{p} = £24.75$

$$a^2 + b^2 = c^2$$





How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

GLUE HERE



Year 11 Mathematics (Higher): Low Stake Test scores (Spring)



Topics	Date	Score
Mixed numbers (4 operations), Negative Indices, Fractional Indices, Surds, Adding and Subtracting algebraic fractions. Cumulative Frequency, Box Plots, Histograms, Direct Proportion and Inverse Proportion.		
Vectors, Iteration, Simultaneous linear equations, Equation of a line, Functions, Solving Simultaneous Equations Graphically, Algebraic Proof, Compound Interest and Reverse Percentages.		
Pythagoras, Trigonometry, Area of a segment, Sine Rule, Cosine Rule, Simultaneous Equations (Quadratic), Completing the Square, changing the subject of a formula, Quadratic formula and quadratic nth term.		
Mixed numbers (4 operations), Negative Indices, Fractional Indices, Surds, Adding and Subtracting algebraic fractions. Cumulative Frequency, Box Plots, Histograms, Direct Proportion and Inverse Proportion.		
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