



Knowledge Organiser: Year 10 (F) - HT1

The square of a number

To square a number, we just multiply it by itself.

For example, to square 5 we multiply 5 by 5. We get an answer of 25.

We can use mathematical notation to save time when squaring numbers. Instead of writing the square of 5 – or 5 squared – we can instead write 5^2 .

Example 1

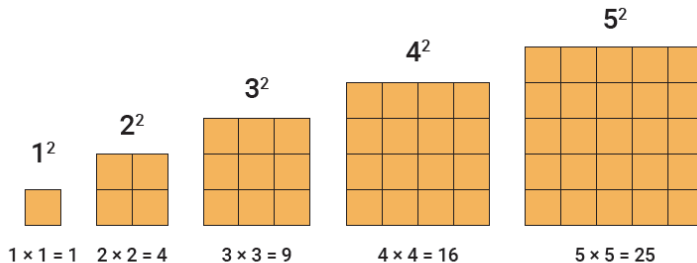
Find the value of the square of 8.
 $8^2 = 8 \times 8 = 64$

Example 2

What is the value of 3^2 ?
 $3^2 = 3 \times 3 = 9$

The answer we get when we square a whole number is called a square number. The first 5 square numbers are 1, 4, 9, 16 and 25.

Square numbers are ones that you can arrange into a square.



Multiples

Multiples are all of the numbers in the times table.

E.g., the multiples of 6 are...

6, 12, 18, 24, 30 ...

Lowest common multiple (LCM)

The lowest number to appear in both lists of multiples.

E.g., the LCM of 6 and 9 is 18.

Multiples of 6 → **6, 12, 18, 24, 30 ...**

Multiples of 9 → **9, 18, 27, 36, 45 ...**

The cube of a number

A cube number is a number multiplied by itself three times. This can also be called a number cubed.

For example, to calculate the value of 2 cubed we multiply 2 by 2 by 2. We get an answer of 8.

We can also use mathematical notation to save time.

For example, instead of writing the cube of 5, or 5 cubed, we can write 5^3 .

Example 1

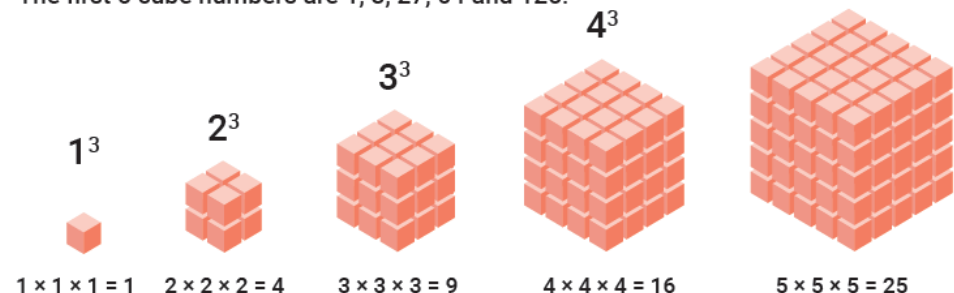
Find the value of the cube of 3.
 $3^3 = 3 \times 3 \times 3 = 27$

Example 2

Find the value of 4^3 .
 $4^3 = 4 \times 4 \times 4 = 64$.

The answer we get when we cube a number is called a cube number.

The first 5 cube numbers are 1, 8, 27, 64 and 125.



The square root of a number

Finding the square root of a number is the **inverse** of squaring a number.

For example, 5 squared is 25. The square root of 25 is 5.
6 squared is 36. The square root of 36 is 6.

We can also use the square root symbol to save time.
For example, instead of writing the square root of 36, we write $\sqrt{36}$.

The cube root of a number

Finding the cube root of a number is the inverse of cubing a number.

For example, the cube root of 125 is 5 because 5 cubed is 125.
The cube root of 8 is 2 because 2 cubed is 8.

We can also use the cube root symbol to save time.
For example, instead of writing the cube root of 8, we can write $\sqrt[3]{8}$.

Multiplying numbers with powers

If we consider $4^3 \times 4^2$ then this gives $(4 \times 4 \times 4) \times (4 \times 4) = 4^5$

Therefore, when we multiply the same base number the powers are added.

$$a^m \times a^n = a^{m+n}$$

E.g., simplify

1) $3^4 \times 3^3 = 3^{3+4} = 3^7$

2) $2^{-2} \times 2^6 = 2^{-2+6} = 2^4$

Dividing numbers with powers

If we consider $7^5 \div 7^2$ then this gives

$$\frac{\cancel{7} \times \cancel{7} \times 7 \times 7 \times 7}{\cancel{7} \times \cancel{7}} = 7^3$$

* Remember $\frac{7}{7} = 1$

Therefore, when we divide the same base number the powers are subtracted.

$$a^m \div a^n = a^{m-n}$$

E.g., simplify

1) $8^7 \div 8^5 = 8^{7-5} = 8^2$

2) $5^3 \div 5^{-6} = 5^{3-(-6)} = 5^9$

Raising a power to a power

If we consider $(6^3)^5$ then this gives

$$6^3 \times 6^3 \times 6^3 \times 6^3 \times 6^3 = 6^{3+3+3+3+3} \text{ or } 6^{3 \times 5} = 6^{15}$$

Therefore, when we raise a power to another power the powers are multiplied.

$$(a^m)^n = a^{mn}$$

E.g., simplify

1) $(4^5)^2 = 4^{5 \times 2} = 4^{10}$

2) $(9^3)^4 = 9^{3 \times 4} = 9^{12}$

Powers that are 0

If we consider $5^4 \div 5^4$ then rules of indices tell us that

$$5^4 \div 5^4 = 5^{4-4} = 5^0.$$

If we look at the division like this

$$\frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}} = 1, \text{ then this shows that } 5^0 = 1.$$

Therefore, any number to the power of 0 will always equal 1 $a^0 = 1$

Using more than one rule

E.g., simplify

$$1) \frac{3^5 \times 3^6}{3^7} = \frac{3^{11}}{3^7} = 3^4$$

$$2) \frac{(8^3)^4}{8^{15} \times 8^{-3}} = \frac{8^{12}}{8^{12}} = 8^0 = 1$$

Multiplication

1) $(5 \times 10^4) \times (3 \times 10^2) = 5 \times 3 \times 10^4 \times 10^2$
 $= 15 \times 10^{4+2}$
 $= 15 \times 10^6$
 $= 1.5 \times 10^7$

2) $(6.4 \times 10^5) \times (2 \times 10^{-3}) = 6.4 \times 2 \times 10^5 \times 10^2$
 $= 12.8 \times 10^{5+(-3)}$
 $= 12.8 \times 10^2$
 $= 1.28 \times 10^3$

Division

3) $(2 \times 10^9) \div (8 \times 10^6) = 2 \div 8 \times 10^9 \div 10^6$
 $= 0.25 \times 10^{9-6}$
 $= 0.25 \times 10^3$
 $= 2.5 \times 10^2$

4) $(9 \times 10^{-4}) \div (3 \times 10^7) = 9 \div 3 \times 10^{-4} \div 10^7$
 $= 3 \times 10^{-4-7}$
 $= 3 \times 10^{-11}$

Addition

5) $(5 \times 10^4) + (6.2 \times 10^3) = 50\,000 + 6\,200$
 $= 56\,200$
 $= 5.62 \times 10^4$

6) $(3.2 \times 10^{-4}) + (5.4 \times 10^{-3}) = 0.00032 + 0.0054$
 $= 0.00572$
 $= 5.72 \times 10^{-3}$

Subtraction

7) $(7.8 \times 10^4) - (5 \times 10^2) = 78\,000 - 500$
 $= 77\,500$
 $= 7.75 \times 10^4$

8) $(2.4 \times 10^{-2}) - (1.8 \times 10^{-3}) = 0.024 - 0.0018$
 $= 0.0222$
 $= 2.22 \times 10^{-2}$

Check that you:

• are familiar with powers of 10

$$10^1 = 10 \quad 10^{-1} = 0.1$$

$$10^2 = 100 \quad 10^{-2} = 0.01$$

$$10^3 = 1\,000 \quad 10^{-3} = 0.001$$

$$10^4 = 10\,000 \quad 10^{-4} = 0.0001$$

$$10^5 = 100\,000 \quad 10^{-5} = 0.00001$$

• understand how to $\times$ and $\div$ by 10, 100, 1000 etc.

• remember rules of indices:

$$a^m \times a^n = a^{m+n} \text{ e.g., } 10^3 \times 10^5 = 10^{3+5} = 10^8$$

$$a^m \div a^n = a^{m-n} \text{ e.g., } 10^7 \div 10^4 = 10^{7-4} = 10^3$$

Remember to write your final answer to calculations in standard form, i.e., a number between 1 and 10 multiplied by a power of 10.

Notice in examples 1 and 2, to write the answers in standard form, the number is $\div 10$ (made ten times smaller) therefore the power of ten must be increased by 1 (made ten times bigger). In example 3, the number is $\times 10$ (made ten times bigger) therefore the power of ten must be decreased by 1 (made ten times smaller).

SIMPLIFYING EXPRESSIONS

We can simplify expressions by collecting similar terms (also called like terms) together.

Example

Simplify $4x + 2 + 2x + 7$.

We can collect the x terms together, which gives $4x + 2x = 6x$, and we can collect the numbers together, which gives $2 + 7 = 9$. The simplified expression is $6x + 9$.

Further examples

- $a + a + a + a = 4a$
- $3a + a + b + 5b = 4a + 6b$
- $9t + 6w - 4t + w = 5t + 7w$

In the first example, there are 4 lots of a , or $4 \times a$. In algebraic terms, we can just write $4a$.

These examples demonstrate how multiplication and division work with algebraic terms.

- $a \times a = a^2$
- $a \times b \times c = abc$
- $3a \times 5b = 15ab$
- $60a \div 3b = \frac{60a}{3b} = \frac{20a}{b}$

If an expression has a set of brackets, we need to expand them first.

Example

$$6a + 3(a + 7)$$

First expand the brackets and then collect like terms.

$$\begin{aligned} 6a + 3(a + 7) &= 6a + 3a + 21 \\ &= 9a + 21 \end{aligned}$$

Expanding brackets

Example 1

Consider $3(2x + 5)$.

This means the same as $3 \times (2x + 5)$.

We have:

$$3(2x + 5) = 3 \times 2x + 3 \times 5$$

This gives a final answer of $6x + 15$.
 $6x$ and 15 are different terms within the expression $6x + 15$.

Expanding brackets involving negative numbers

Care is needed if there is a negative number either inside or outside the brackets.

Example 2

Expand $4(3x - 5)$.

Answer:

$$4(3x - 5) = 4 \times 3x + 4 \times (-5) = 12x - 20$$

Example 3

Expand $-2(x - 6)$.

Answer:

$$-2(x - 6) = (-2) \times x + (-2) \times (-6) = -2x + 12$$

This could also be written as $12 - 2x$.

Example 4

Expand $-(7x - 3)$.

Answer:

Remember that

$-(7x - 3)$ is the same as $-1(7x - 3)$.

$$-(7x - 3) = (-1) \times 7x + (-1) \times (-3) = -7x + 3$$

Adding negative numbers

Adding a negative number is the same as subtracting the positive value of the number.

Examples

$$6 + -4 = 6 - 4$$

$$= 2$$

$$-9 + -5 = -9 - 5$$

$$= -14$$

Subtracting negative numbers

Subtracting a negative number is the same as adding the positive value of the number.

Examples

$$6 - -4 = 6 + 4$$

$$= 10$$

$$-4 - -5 = -4 + 5$$

$$= 1$$

Multiplying negative numbers

When we multiply two numbers:

positive number $\times$ positive number = positive number

positive number $\times$ negative number = negative number

negative number $\times$ positive number = negative number

negative number $\times$ negative number = positive number

Examples

$$-4 \times 7 = -28$$

$$-5 \times -4 = 20$$

Dividing negative numbers

When we divide two numbers:

positive number $\div$ positive number = positive number

positive number $\div$ negative number = negative number

negative number $\div$ positive number = negative number

negative number $\div$ negative number = positive number

Examples

$$-6 \div 2 = -3$$

$$-20 \div -4 = 5$$

Double brackets are trickier than single brackets — this time, you have to multiply **everything** in the **first bracket** by **everything** in the **second bracket**. You'll get **4 terms**, and usually 2 of them will combine to leave **3 terms**. There's a handy way to multiply out double brackets — it's called the **FOIL method**:

- First** — multiply the first term in each bracket together
- Outside** — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)
- Inside** — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)
- Last** — multiply the second term in each bracket together

EXAMPLE Expand and simplify $(2p - 4)(3p + 1)$

$$\begin{aligned}
 (2p - 4)(3p + 1) &= (2p \times 3p) + (2p \times 1) + (-4 \times 3p) + (-4 \times 1) \\
 &= 6p^2 + 2p - 12p - 4 \\
 &= 6p^2 - 10p - 4
 \end{aligned}$$

The two p terms combine together.



Uses of foil: expanding brackets, jacket potatoes...

Always write out **SQUARED BRACKETS** as **TWO BRACKETS** (to avoid mistakes), then multiply out as above. So $(3x + 5)^2 = (3x + 5)(3x + 5) = 9x^2 + 15x + 15x + 25 = 9x^2 + 30x + 25$. (DON'T make the mistake of thinking that $(3x + 5)^2 = 9x^2 + 25$ — this is **wrong wrong wrong**.)

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Factors

Numbers that divide exactly into another number with no remainder.

E.g., the factors of 18 are...

1, 2, 3, 6, 9 and 18

Prime numbers

Prime numbers have only 2 factors, 1 and the number itself.

E.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 ...

Highest common factor (HCF)

The highest number to appear in both lists of factors.

E.g., the HCF of 18 and 45 is 9.

Factors of 18 → 1, 2, 3, 6, 9, 18
 Factors of 45 → 1, 3, 5, 9, 15, 45

Factorising by finding the highest common factor

When every term in an expression has a common factor it is possible to factorise the expression by placing the highest common factor outside of the brackets. This is the largest number or letter (or both) that divides exactly into every term of the expression. There are many methods to factorise, only one of which is given here.

1

Factorise $6x + 18$

1, 2, 3 and 6 are all numbers that divide into both terms, but 6 is the highest common factor therefore this is placed outside of the brackets.

$$6(\quad)$$

To complete the brackets, we must find the terms that multiply with 6 to give $6x$ ($6 \times x$) and to give $+18$ (6×3)

$$\text{Therefore } 6x + 18 = 6(x + 3)$$

2

Factorise $8x^2 - 12x$

1, 2 and 4 are all numbers that divide into both terms in the expression, with 4 being the largest. Both terms are also divisible by x , therefore $4x$ is the highest common factor of $8x^2$ and $-12x$. This is the term placed outside of the brackets.

$$4x(\quad)$$

To complete the brackets, we must find the terms that multiply with $4x$ to give $8x^2$ ($4x \times 2x$) and $-12x$ ($4x \times -3$)

$$\text{Therefore } 8x^2 - 12x = 4x(2x - 3)$$

3

Factorise $6a^2b + 15ac^2$

1 and 3 are the numbers that divide into both terms in the expression, with 3 being the largest. Both terms are also divisible by a , therefore $3a$ is the highest common factor and this is the term placed outside of the brackets.

$$3a(\quad)$$

To complete the brackets we must find the terms that multiply with $3a$ to give $6a^2b$ ($3a \times 2ab$) and $+15ac^2$ ($3a \times 5c^2$)

$$\text{Therefore } 6a^2b + 15ac^2 = 3a(2ab + 5c^2)$$

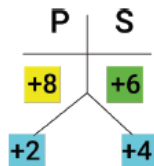
Factorising quadratic expressions

A quadratic expression can take the form $x^2 + ax + b$. There are many possible methods to factorise a quadratic expression, only one of which is given here.

1

Factorise $x^2 + 6x + 8$.

Here you need two numbers with a product of $+8$ and a sum of $+6$.



(If you can't find the numbers immediately, try listing factor pairs of $+8$, namely 1×8 , -1×-8 , 2×4 , -2×-4)

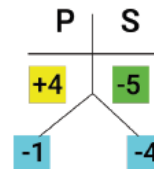
Place the two numbers in brackets like this

$$(x + 2)(x + 4) \quad \text{or} \quad (x + 4)(x + 2)$$

2

Factorise $x^2 - 5x + 4$

Here you need two numbers with a product of $+4$ and a sum of -5 .



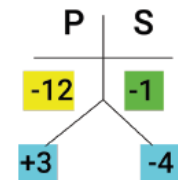
Place the two numbers in brackets like this

$$(x - 1)(x - 4) \quad \text{or} \quad (x - 4)(x - 1)$$

3

Factorise $x^2 - x - 12$

Here you need two numbers with a product of -12 and a sum of -1 .



Place the two numbers in brackets like this

$$(x - 4)(x + 3) \quad \text{or} \quad (x + 3)(x - 4)$$



How do we use Knowledge Organisers in Mathematics

How can you use knowledge organisers at home to help us?

- **Retrieval Practice:** Read over a section of the knowledge organiser, cover it up and then write down everything you can remember. Repeat until you remember everything.
- **Flash Cards:** Using the Knowledge Organisers to help on one side of a piece of paper write a question, on the other side write an answer. Ask someone to test you by asking a question and seeing if you know the answer.
- **Mind Maps:** Turn the information from the knowledge organiser into a mind map. Then reread the mind map and on a piece of paper half the size try and recreate the key phrases of the mind map from memory.
- **Sketch it:** Draw an image to represent each fact; this can be done in isolation or as part of the mind map/flash card.
- **Teach it:** Teach someone the information on your knowledge organiser, let them ask you questions and see if you know the answers.

How will we use knowledge organisers in Mathematics?

Knowledge organisers will be used before I complete a Learning Check or Common Assessment. I will spend part of the lesson looking over each of the key topics of the half term before completing the Learning Check or Common Assessment.

I will also use these at home to complete my own independent learning and revision of these key topics.

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